What is Mathematics Education for?

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It is argued that familiar arguments for the status of mathematics as a major school subject, while retaining validity, need to be subjected to deep critique in relation to the roles played by mathematics in social, economic, and political aspects of our lives. Accordingly, mathematics educators (and mathematicians) should examine their ethical responsibilities in relation to the challenges facing humankind.

Asking Deep Questions

What is mathematics education for? Many answers to these questions have traditionally been, and continue to be, advanced. Standard reasons include the need to produce another generation of scholars to continue developing the discipline of mathematics; the supply of a cadre of scientists and others such as engineers who need strong mathematical competence; as a training in logical thinking and problem solving; as exposure to what is as much a part of cultural heritage as literature or music. All of these, and more, are valid, but a deeper analysis is required.

We need to avoid what Ubiratan D'Ambrosio calls "the trap of the same". Computers have changed the nature of mathematics. Technological developments have radically altered the flow of information and communication. Concurrently, the sheer amount of systematized mathematics has increased hugely, with the result that designing a curriculum is no longer easy (if it ever was) and extremely difficult choices have to be made. However, these changes, significant though they are, are largely internal to mathematics-as-a-discipline in its relationship with mathematics-as-a-school-subject. There are much larger questions that are raised by considering the roles of mathematics in social and political contexts.

Mathematics as Cultural Achievement

In my opinion, students should learn more than they typically do about the intellectual and social history of mathematics. Tracing the historical development of the concept of number across cultures, for example, is both interesting in its own right and offers insights into the conceptual obstacles that learners face. A balanced acknowledgment of the contributions of many cultures to the development of mathematics also serves to counter the pervasive Eurocentrism in standard Western accounts of the history of mathematics. In the extreme case, the narrative presented is that mathematics proper began with the Greeks, disappeared for many centuries, re-emerged in Renaissance Italy, and developed from there to its contemporary greatness. Such a view ignores the achievements in formal mathematics in India, China, Arabia, South America, and many other cultures, as has been documented by George Joseph (1992).

George and his colleague Almeida are involved in a fascinating and ongoing debate about the extent to which the mathematical achievements of the Kerala school in the 14th to 16th centuries might have been transmitted to Europe and laid the foundation for the development of calculus by Newton and Leibnitz (Almeida & George, 2007).

A rich history of mathematics also goes beyond a mere catalogue of individuals, theorems, and conceptual growth, to consider the social, cultural, historical, and political situatedness of mathematics as a human activity. A fine example is Hacking's (1975, 1990) analysis of the interplay between the mathematical theory of probability and conceptualizations of human society across several centuries. (An interesting comment in Hacking (1975, p. 8) is that "it is reasonable to guess that a good deal of Indian probability lore is at present unknown to us".)

Below the Tip of the Pyramid

Imagine a pyramid representing the population of school students studying mathematics. The tip of the pyramid represents the small percentage of students who continue to learn, use, and possibly research, high-level mathematics in their careers. For the large majority below the tip, it is appropriate to consider what might be a suitable mathematics education to prepare them for intellectual fulfillment and as future citizens. In this regard, it is surely essential to make students aware of the implications of mathematisation in their societies and also the complementary aspect of "demathematisation", by which is meant the invisibility of the mathematics that has been incorporated into physical and cultural artifacts (Keitel, Kotzmann, & Skovsmose, 1993; Gellert & Jablonka, 2007). Mathematics "formats" society (as does language, of course), in the sense introduced by Skovsmose and Keitel, and discussed in Davis and Hersh (1986). In a kind of feedback loop, the modelling of social aspects of reality itself changes that reality (Skovsmose, 2006).

Does every citizen-to-be need to learn substantial amounts of mathematics (formal algebra, in particular), as is declared both possible and essential in many contemporary policy documents, particularly in the United States? Here is the view of one mathematician (Davis, 1999):

> How much fluency is sufficient? As regards the average person living in a highly mathematized, but chipified, multimedia technological civilization, I think that rather less is required on a day-to-day basis than is often argued in official recommendations.
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> What is necessary is to teach enough so that the commonplace diurnal mathematical demands placed on the population are readily fulfilled. What is also necessary is to infuse sufficient mathematical and historical literacy that people will be able to understand that the mathematizations put in place in society do not come down from the heavens: that they do not operate as pieces of inexplicable ju-ju, that mathematizations are human cultural arrangements and should be subject to the same sort of critical evaluation as all human arrangements.
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> At the risk of sounding like a traitor to my profession, I would say that high school algebra or beyond is not necessary to achieve this goal. A bit of elementary probability would be a good thing, as would knowledge of the major time-variation templates: linear, exponential, periodic. A mandatory short course called Skepticism 101 would help students hone critical skills in all areas. I and many others would like to see students' ability to be critical strengthened greatly; I know of some teachers who have been working toward just this goal, which may be more important than finding the volume of a cone by rote.

An alternative to the view that all students should be expected to learn considerable amounts of formal mathematics is that mathematics education should be directed more towards providing future citizens with tools to evaluate critically, and act upon, issues of importance in their personal lives and the lives of their communities. Such an approach would address the concerns of many mathematics educators that mathematics education is not connected with the lived experience of students. For example, Fasheh (2000, p. 5) declared that: " I cannot subscribe to a system that ignores the lives and ways of living of the social majorities in the world; a system that ignores their ways of living, knowing and making sense of the world."

Training in Logical Thinking – Or Training in Simplistic Thinking?

Here I present some threads of an argument to the effect that typical school mathematics education provides a training in forms of thinking that are simplistic in relation to the complexity of even the physical world, and more so in relation to social phenomena that are increasingly being modeled mathematically (Davis & Hersh, 1986). Here a distinction may be made between the idealization that occurs in all mathematical modelling that is mindfully done by mathematicians and scientists aware of the implications, and simplistic assumptions about direct relationships between aspects of reality and mathematical structures.

By way of example, consider the concept of function, central to algebra and calculus. A function models a relationship between two variables that is deterministic. To an extent, that makes it useful for the modeling of some simple physical phenomena. However, as the mathematics of, for example, probabilistic, multivariate, and nonlinear systems develops, and with the considerable change brought about by the availability of massive computational power, the range of what can be modeled is expanding very fast and entering new realms of complexity.

In terms of mathematics education, one necessary change is to counter the impression that real-world phenomena can always be unproblematically mapped onto mathematical structures. By way of example, when students aged 10-13 were posed the following as an item in a test using word problems: "John's best time to run 100 meters is 17 seconds. How long will it take him to run 1 kilometer?", the percentage of students tested in several countries who showed any awareness whatsoever that a directly proportional answer is problematic, in relation to the reality of the situation described, ranged from 0% to 7% (Verschaffel, Greer, and De Corte, 2000, p. 25). Similar results were found for a range of items (and have been confirmed in replications across the world). The findings for this particular item fit within a general pattern investigated in a program of research that shows how the tendency to assume linearity and proportionality in cases where they are not appropriate is extremely strong and pervades all branches of mathematics (De Bock, 2002; Van Dooren, 2005). One solution proposed in Verschaffel et al. (2000) is to emphasise much more strongly the principles of mathematical modeling, and to do so from an early age (Usiskin, 2007).

As indicated in the quotation from Davis above, another curricular shift that is necessitated by both developments within mathematics and the increasing modeling of the social and physical worlds is to teach the core ideas of statistical analysis and probabilistic modeling. As I write this paper, a presidential election campaign is happening in the US, in which polls play a prominent role. Statistical concepts such as "margin of error ±3%" are frequently used, yet very few citizens have a basic understanding of what such phrases mean. An analysis of reporting on estimates of civilian deaths during the Iraq War (Greer, 2008a) illustrates not only political control and ideological distortion of the available information but also profound lack of understanding of the most basic aspects of sampling methodology, not just on behalf of ordinary citizens, but also in what should be informed media commentaries.

Nationalistic Mathematics or Mathematics in the Service of Humankind?

In the contemporary world, it is has become commonplace to speak of mathematics and science education as essential to a nation to maintain or improve its economic position in a competitive world. Such a nationalistic perspective is seen in essentially every major country, but reaches an extreme in the United States, where there is currently great anxiety that advances in other parts of the world, notably India and China, will threaten American global supremacy. This anxiety is expressed in reports with alarmist titles such as *Rising Above the Gathering Storm* (National Academies, 2006) and in the American Competitiveness Initiative (www.ed.gov/about/inits/ed/competitiveness/index.html?src=pb). In the recently released final report of the National Mathematics Advisory Panel (United States Department of Education, 2008, p. xi), it is stated that:

> Much of the commentary on mathematics and science in the United States focuses on national economic competitiveness and the economic well-being of citizens and enterprises. There is reason enough for concern about these matters, but it is yet more fundamental to recognize that the safety of the nation and the quality of life – not just the prosperity of the nation – are at issue (see Gutstein, in press).

Rather than mathematics being used for narrow nationalistic goals, as part of the military/industrial/academic complex, a different choice is possible – using mathematics to analyse and work on issues of social justice. Numbers can tell a story very starkly. There is a fine book for children (and adults) called "If the world were a village" (Smith and Armstrong, 2002) which provides proportional statistics that would apply if the world were a village of 100 people. For example, 61 are from Asia, 13 from Africa, 12 from Europe, 8 from South and Central America and the Caribbean, 5 from Canada and the United States, 1 from Oceania. Only 24 always have enough to eat, while 60 are always hungry (26 being severely undernourished), and 16 go to bed hungry at least some of the time. The richest 20% each have more than $9000 a year, the poorest 20% less than $400. A more sophisticated measure of wealth dispersion is the Gini index, determined by graphing percentage of income against percentage of households and finding the area between this graph and the straight line representing total equality. On this index, a score of 0 would represent total equality and a score of 100 total inequality. According to recent data from Human Development Reports (hdrstats.undp.or/indicators/147.html) the worst values are found in African and South American countries (over 50 in many cases). The lowest values (around 25) are found in Scandinavian countries. The values for India, Pakistan, and Bangladesh are 37, 33, 31 respectively. The value for the United States is 41. A vital aspect of teaching the mathematics behind the GINI index is that the interpretation is by no means straightforward (the Wikipedia entry: en.wikipedia.org/wiki/Gini_coefficient) does an excellent job in pointing out its weaknesses as a measure of wealth dispersion).

The Brazilian scholar and peace activist, Ubi D'Ambrosio (2003) has called on mathematicians and mathematics educators to accept their ethical responsibilities:

> It is widely recognized that all the issues affecting society nowadays are universal, and it is common to blame, not without cause, the technological, industrial, military, economic and political complexes as responsible for the growing crises threatening humanity. Survival with dignity is the most universal problem facing mankind.
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> Mathematics, mathematicians and mathematics educators are deeply involved with all the issues affecting society nowadays. But we learn, through History, that the technological, industrial, military, economic and political complexes have developed thanks to mathematical instruments. And also that mathematics has been relying on these complexes for the material bases for its continuing progress. It is also widely recognized that mathematics is the most universal mode of thought.
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> Are these two universals conflicting or are they comple-

> mentary? It is sure that mathematicians and math educators, are concerned with the advancement of the most universal mode of thought, that is, mathematics. But it is also sure that, as human beings, they are equally concerned with the most universal problem facing mankind, that is, survival with dignity.

Education is Politics

The Brazilian liberatory educator, Paulo Freire (1987, p. 46) declared:

> This is a great discovery, education is politics! After that, when a teacher discovers that he or she is a politician, too, the teacher has to ask, What kind of politics am I doing in the classroom?

Freire's statement is about education in general, but there are major reasons why it is particularly applicable to mathematics, especially because of the extent to which mathematics influences modern societies, often in ways that are hidden (Davis & Hersh, 1986; Skovsmose, 2006). However, as Apple (2000, p. 243) has pointed out:

> It is unfortunate but true that there is not a long tradition within the mainstream of mathematics education of both critically and rigorously examining the connections between mathematics as an area of study and the larger relations of unequal economic, political, and cultural power.

The choice for individual intellectuals was stated thus by Sinha (2000, p. 4193):

> From the time of their emergence the options lying before the intellectuals are two:
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> (i) to operate as functional intellectuals who excel in providing legitimacy to the order of things because they neither question nor raise any doubt [about] the existing order;
>
> (ii) to establish themselves as oppositional-critical intellectuals who through applying their critical faculty [seek] to struggle for a better society ...

The Final Report of the National Advisory Panel on Mathematics (United States Department of Education, 2008) that was established by President Bush offers a clear example of the first option (Greer 2008b). A search reveals no occurrences of the following words: cultural, ethical, demography/demographic, equality (except in the mathematical sense), equity, social justice, political/politics. It does not even mention the No Child Left Behind legislation that has had such a dominant political influence in recent years in the United States as a massive exercise in social engineering.

The relationships between three aspects of mathematics – mathematics as a discipline, mathematics as a school subject, and mathematics as a part of people's lives – need serious analysis. To promote a vision of what mathematics education should be for, mathematics educators (and mathematicians) need to engage politically.

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