Unmasking the Gods: Clarifying Social Construction Theses in Mathematics Education

Jagdish Madnani

Sahyadri School, Pune, India

The phrase 'social construction' has raised hackles in academic debates and has rapidly gained currency among theorists of all stripes. Unfortunately, as almost always happens, once a phrase is used too often, it begins to lose its impact and its original meaning-in-use. One of the goals of this paper is to reinvigorate this metaphor by trying to look at some of the roles it has played in the various debates. Another goal is to try to clear the air regarding its use in mathematics. Although mathematical development is often seen as dialogical and historically contingent, it is a discipline that hides its nature under a monological mask that claims to be fixed, monolithic and eternally true. By explaining the use of this phrase in recent writing relating to the philosophy of mathematics and mathematics education, I hope to re-illuminate the contingent nature of the discipline and the mathematics classroom

"In mathematical construction we are, as it were, gods." – Salomon Maimon

Second Life for a Dying Metaphor

The idea of social construction is everywhere. As Hacking (1999) indicates lucidly, this idea has proliferated and replicated itself to the extent that if it were like cancerous cells, "death would be immediate" (p. 3). Without trying to fix the meaning of this phrase, he advises us that it might be useful to question the *overall purpose* of using 'social construction' as a prefix, in the first place, in the various debates. In other words, by understanding its usage in the various debates, we might be better placed to evaluate its implications when it arises in a different field. In trying to answer this question, we might also be able to write a better obituary and cast about for a newer, fresher idea (or phrase) if its use is not warranted in particular areas of study. At one level this might seem like a trivial, purely linguistic enterprise, since one is apparently arguing about 'just a phrase'. It will be my endeavor to try to show how 'just a phrase' can significantly muddle the conversation, specifically in the case of mathematics education, due to certain previously developed ideas in the discipline of mathematics that have the potential of causing theorists and practitioners to talk past each other.

So what is implied in the use of the phrase 'social construction'? By referring to its usage in different contexts and debates (for example, the debate around the social construction of gender), Hacking (1999) is very illuminating in trying to trace a pattern of its implied meanings. One of the main reasons for its usage is for the purpose of "raising consciousness" (p. 6), by indicating that the 'object' that is under consideration is not inevitable:

Social construction work is critical of the status quo. Social constructionists about X tend to hold that:

1. X need not have existed, or need not be at all as it is. X, or X as it is at present, is not determined by the nature of things; it is not inevitable.

Very often they go further, and argue that:

2. X is quite bad as it is.
3. We would be much better off if X were done away with, or at least radically transformed.

A thesis of type (1) is the starting point: the existence or character of X is not determined by the nature of things. X is not inevitable. X was brought into existence or shaped by social events, forces, history, all of which could have been different. Many social construction theses at once advance to (2) and (3), but they need not do so. One may realize that something, which seems inevitable in the present state of things, was not inevitable, and yet is not thereby a bad thing. But most people who use the social construction idea enthusiastically want to criticize, change, or destroy some X that they dislike in the established order of things. (p. 7)

What is achieved by such rising of consciousness? Hacking believes that in some cases it can lead to liberation, by denying inevitability. For example, what could a thesis titled "The Social Construction of Woman Refugees" (Moussa, 1992, as cited in Hacking, 1999) possibly hope to achieve when it is quite clear that women refugees are the product of social forces, and almost no one would claim that the existence of women who have to flee their countries is an agreeable situation? What is gained by prefixing the thesis with "The Social Construction of…"? Hacking claims that it is the idea or the *category* of 'women refugees', which is socially constructed. It is not the actual women under consideration, who are obviously the result of social forces working on them and which is why they choose to or are forced to flee, but it's the creation or 'construction' of this *classification* that is at issue here. But why is the construction of this category problematic? Hacking claims that such ideas or classifications are of the *interactive* kind. In other words, such categorizations interact with the actual persons being categorized, who in turn interact with the categorization itself:

> People of these kinds can become aware that they are classified as such. They can make tacit or even explicit choices, adapt or adopt ways of living so as to fit or get away from the very classification that may be applied to them. These very choices, adaptations or adoptions have consequences for the very group, for the kind of people that is invoked. The result may be particularly strong interactions. What was known about people of a kind may become false because people of that kind have changed in virtue of what they believe about themselves. I have called this phenomenon *the looping effect of human kinds* (Hacking, 1999, emphases original).

In this sense, it is possible for the actual women (or those involved with them) to realize the contingent nature of their classification and what that entails. The first step in politicizing the situation of women refugees in order to bring about a transformation in their situation (if indeed that is the purpose) would be to realize the contingent, historical, non-essential nature of their classification itself.

In the case of the thesis "The Social Construction of Quarks" (Pickering, 1986, as cited in Hacking, 1999), what is claimed to be socially constructed? And what is achieved by inserting the prefix 'social construction'? Surely, quarks are not human kinds and thus the phenomenon of looping would not be applicable here: if quarks were classified in some other way, it would not make any difference, as far as we know to the quarks itself (themselves!). And quite clearly, quarks would not exist without a social organization of physicists, particle accelerators and so on. According to Hacking, this is *not* the reason for prefixing 'social construction' before quarks, in Pickering's thesis. It is to indicate that there *could* have been many possible trajectories to high-energy physics, and one of those trajectories led to the 'construction' of quarks. One objective in using the phrase 'social construction' here would be to show that the path to the development of the idea of quarks was not unique, that there could have been many different successful physics that could have evolved, and not all of these would have necessarily given birth to quarks. Thus, the overall aim of this thesis is to deny inevitability and affirm contingency of the object under consideration – in this case, quarks.

Quite clearly, these examples are from opposite ends of the spectrum: in one case a particular type of human kind (the category 'woman refugee') is being constructed and in the other case the thesis concerns an object from the physical sciences (quarks). The contentious issues in the latter case have to do with contingency, metaphysical nominalism and the stability of the 'object' under consideration used as evidence of its inevitability. The context within which the phrase 'social construction' is used, matters heavily and thus it becomes important to be clear about *what exactly* is claimed to be socially constructed: is it a kind of person, or is it an object that due to the reasons mentioned above seems to be inevitable, but maybe on closer consideration is not? These are important questions to answer before one takes positions in any debate around social construction, although it is clear that a clear binary distinction between an idea and the object it refers to may not be possible, or at any rate the subject of yet another debate! And yet, for the purpose of saying something meaningful about social construction theses, an artificial distinction is made by Hacking between 'things-in-the-world', or 'objects' (such as people, practices, actions and experiences among others), 'ideas' (including conceptions, beliefs, theories and for our purposes, classification or categorization of people, although groups of such classified people would come under 'objects'), and 'elevator' words (such blatantly circularly defined words as truth, reality, knowledge, facts) (Hacking, 1999, pp. 21-23). While there is an awareness of considerable slippage in this kind of artificial category-making, Hacking coins these labels mainly to bring out the distinction between theses that deal with, for example, 'woman refugees' on the one hand and 'quarks' on the other – in one case the thesis is dealing with the category (or idea), and in the other, it is about an object.

Mathematical Constructivism

Within the philosophy of mathematics, Constructivism is a school of thought that aserts that it is necessary to find (or construct) a mathematical object in order to prove that it exists. Incidentally, this school of thought has linkages to another school of thought called Intutionism (a variant of Kant's intuitions of space and time), which itself is historically associated with mathematicians like Brouwer and Bishop. For constructivists, the commonly used indirect proof (reductio ad absurdum), where one asumes that an object does not exist, and then derives a contradiction from that assumption by a series of logically connected steps, thus proving the existence of the object, is not considered valid. A constructive proof, in contrast, is a method of proof that clearly demonstrates the existence of a mathematical object by creating or providing a clear method for creating such an object. Another noteworthy aspect of constructive mathematics is its denouncement of the Law of the Excluded Middle, which simply states that a proposition is either true or false-there is no middle ground, even if one does not at present know the truth-value. Quite clearly, the methods adopted by this school have been controversial:

> Traditionally, mathematicians have been suspicious, if not downright antagonistic, towards mathematical constructivism, largely because of the limitations that it poses for constructive analysis. These views were forcefully expressed by David Hilbert in 1928 when he wrote in *Die Grundlagen der Mathematik*, "Taking the principle of excluded middle from the mathematician would be the same, say, as proscribing the telescope to the astronomer or to the boxer the use of his fists". (http://en.wikipedia.org/wiki/Constructivism_(mathematics))

And here is the redoubtable Hardy (1993):

> Reductio ad absurdum, which Euclid loved so much, is one of a mathematician's finest weapons. It is a far finer gambit than any chess gambit: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game. (p.34)

It is important to keep in mind the full import of the limitations that the constructivist school imposes on mathematicians and the hostility that this school of thought faces from some mathematicians if one is to understand the bewilderment that a phrase such as 'social construction' might evince from the community of mathematicians. It is possible that at least a part of this confusion that might be caused by attaching the prefix 'social' in front of 'constructivism'. As Hacking (1999) says:

> If we left "constructivism" to mathematics, we would avoid the confusion invited by a title such as *Social Constructivism as a Philosophy of Mathematics* (Ernest, 1998), which suggests, to anyone who knows anything about mathematical constructivism, something like a social variant of Brouwer's program (a rather incoherent idea). It would have been better, I think, to speak of social constructionism as a philosophy of mathematics, a philosophy that would presumably maintain that in some sense mathematical objects, such as numbers, and mathematical facts-theorems are social constructs. That would be analogous to constructionism about the natural sciences, although the arguments would presumably be different. (p. 48)

Regardless of such quibbling, what is the major interpretation of the phrase 'social constructivism' within the realm of mathematics? It is worth quoting Ernest (2004):

> Social Constructivism claims that the concepts, definitions, and rules of mathematics (including rules of truth and proof) were invented and evolved over millennia. Thus mathematical knowledge is based on contingency, due to its historical development and the inevitable impact of external forces on the resourcing and direction of mathematics. (p. 25)

Modern Conception of Proof – Historical Accident?

One might well ask – are there aspects within the discipline itself that might be considered, at large, to be inevitable, but which on closer examination turn out to be the result of some historical accident? In a sense, without getting into the timeless and slippery debate between Platonic and humanistic conceptions of mathematical objects, can one find *something central and meaningful* within the field of mathematics that need not have been the way it is, and yet has been elevated to the status of inevitable and ahistoric?

Hacking (2002) considers the modern conception of proof in mathematics and how one can trace its origins in the seventeenth century. Unlike Euclidean proofs which seemed to rely at least to some extent on the content of the proofs (the actual geometrical entities and their relationships) and not on the form, or grammar of the proof itself, Hacking (2002) shows how our modern conception of mathematical proof is very different and actually resembles Leibniz's ideas during the seventeenth century:

> In saying that Leibniz knew what a proof is, I mean that he anticipated in some detail the conception of proof that has become dominant in our century…A proof, thought Leibniz, is valid in virtue of its form, not its content. It is a sequence of sentences beginning with identities and proceeding by a finite number of steps of logic and rules of definitional substitution to the theorem proved. (p. 201)

Interestingly, according to Hacking (2002), it was a creation of Descartes, analytic geometry, which in a sense, by stripping geometry of its content, aided, unintentionally perhaps, Leibniz's conception of proof. It is also interesting that these two great figures of mathematics had very different ideas about the relationship between truth and proof: "Leibniz thought that truth is constituted by proof. Descartes thought proof irrelevant to truth" (Hacking, 2002, p. 204). Another very important historical event that was responsible for the emergence of this new formulation of proof was the epistemological upheaval that was taking place in science in the seventeenth century. The realization that the new science was no longer "Aristotelian knowledge or *scientia*" (Hacking, 2002, p. 211), caused a vacuum, and in a sense, Leibniz's conception of formal proof which was detached from 'eternal truths,' moved to fill the empty space that was largely created by the conditions prevalent at that time. In other words, not only did Leibniz's conception of proof not exist in any meaningful way before the seventeenth century, it was contingent on this vacuum that was created. Adding weight to this thesis is Popkewitz (2004):

> Hacking argues that mathematics embodies different ways of thinking about and creating new objects. Each style of reasoning in mathematics, Hacking continues, opens up different objects for scrutiny and provides classificatory schemes by which lives are experienced, truths authenticated, and futures chosen…The various styles of reasoning introduce different registers of debates about the ontological status of the objects "seen" as true. Approaching science and mathematics as fields of cultural practices that construct their objects and truth statements, he argues, is a way out of the controversies that divide philosophy and education into realist and anti-realist camps. It is a way to overcome the unproductive separation of epistemology and ontology and the division between subjectivist and objectivist worldviews. (p. 20, original emphases)

Thus the modern conception of proof that is now formalized and engraved on mathematics has a memory. It is true that the necessary relationship between axioms and theorems is not contingent, yet any particular style of reasoning in mathematics does have a contingent, historical basis – and thus in Hacking's (1999) terms, as mentioned above, the modern conception of mathematical proof is socially constructed. Ernest (2004) has a very similar conception:

> Much of mathematics follows by logical necessity from its assumptions and adopted rules of reasoning, just as moves do in a game of chess. Once a set of axioms and rules has been chosen (e.g., Peano's axioms or those of group theory), many unexpected results await the research mathematician. This does not contradict the skeptical epistemology of social constructivism for none of the rules of reasoning and logic in mathematics are themselves absolute. (p. 25)

School Mathematics as a Social Construction

According to Popkewitz (2002), there is little or no connection between academic disciplines and their school avatars, due to the transmogrification, or "alchemy" that occurs when the knowledge of an academic discipline moves to school. Although this alchemy is an unavoidable fact of schooling, Popkewitz (2002) claims:

> In mathematics education, the alchemic transformation can be explored further. On the surface, the discussion is about teaching children about mathematics. Teacher education research focuses on the content and structure of teachers' knowledge, such as learning about the development of children's mathematical thinking and problem solving. Best practices in instruction, for example, are to teach problem solving in algebra and geometry and children's learning multiple solutions in making conjectures and justifications. The evidence of research is the identification of children's thinking processes or the teacher's pedagogical content knowledge that furthers the problem solving. However, the problem solving of mathematics education is a fiction of the alchemy. The problem solving of mathematics is not some universal system of rules about conjectures and justification but an academic field of cultural practices concerning norms of participation, truth, and recognition that change over time (see, e.g., Van Bendegem, 1996). The research on mathematics education focuses on psychological theories of problem solving and the management practices related to the classroom of children's learning. The principles selected as mathematics concepts conform and translate into the expectations of pedagogy as studies of children's thinking. The evidence of learning mathematics is formed through the lenses of cognitive psychology, notions of child growth and development, and sometimes social-psychological concepts, such as situated learning. Expected teacher performance in mathematics education is to develop instruction that captures children's intuitive understanding of conventional mathematics concepts. (p. 263)

Additionally, Popkewitz (2002) also claims that as disciplinary knowledge moves into schools, it crystallizes, thus implying a fixed, unalterable, body of knowledge. Although mathematics as a discipline is characterized as a cultural practice that is related to the changing norms and practices of the community of mathematicians, in reform documents, mathematics is assumed to be stable and unchanging. As Popkewitz (2004) suggests, maintaining this conception of mathematics allows for the 'proper' administration of the child in the classroom, through associated devices and pedagogical 'eyes'. The politics of reform and indeed of schooling in general, combine with psychological and social inscriptions to 'create' a particular kind of school mathematics with its own logic and driving force, not entirely congruent with the practices and driving force of the academic discipline itself. In this sense, mathematics in schools can be considered to be socially constructed.

In fact, it is through "inscription devices", as claimed by Popkewitz (2004), that the child in the mathematics classroom is "fabricated" too. One example of an "inscription device" propagated through standards-based reform documents in the US is the 'problem-solving child'. Pedagogical research in mathematics education is not entirely a description of classroom practices or even about the content of mathematics. It plays a role in actually constructing or 'fabricating' certain *human kinds* (Hacking, 1999, 2002) upon whom the gaze of the researcher is then directed, in order to reveal the subject as transparent. "Fabrication directs attention to how linguistic categories and distinctions of educational research are both fictions and creators of "things". As linguists say, language functions simultaneously to construe and to construct" (Popkewitz, 2004, p. 13). Just as in the case of the 'woman refugee' (Hacking, 1999), the newly minted 'problem-solving child' is an *interactive* kind of categorization. In other words, the actual, real flesh and blood child who is considered to fall within that category is then in an interactive relationship with the 'idea' of this child and the role of the teacher/education researcher is to save or "rescue" the child from being one who "is left behind" (Popkewitz, 2004). By 'constructing' such a category, it becomes possible to measure, govern and administer the mathematical 'well being' of children in the classroom who fall (or indeed are *made* to fall) under such a category. Indeed, according to Popkewitz (2004), it is not only the mathematical well being that is the focus of such inscriptions – ethical and political considerations having to do with the creation of 'proper' citizens in an ever-changing world is a very important aspect of the 'translation' that the pedagogical 'eye' of psychology plays in transmogrifying mathematics into mathematics education. While carefully reviewing certain reform documents in the US, Popkewitz (2004) looks askance at the use of phrases such as "discourse communities of learning", which although at first might seem to mimic communities of mathematicians, and yet on closer inspection have little to do with mathematics or its practice:

> This "doing" of mathematics is not only about cognitive learning but also about one's moral being and involvement in the world…[But as] the discourses of mathematics education are examined more closely, it is not mathematics that is the site of intervention. It is the governing of the child as a moral agent. The homage paid to the "doing" of mathematics is quickly transmogrified into sociopsychological conceptions of child development. (p. 10)

In a study of representations of students in the mathematics classrooms, within the mathematics education literature, Valero (2004) finds the construction of a 'fictional' cognitive student, almost devoid of any socio-cultural context:

> If I were asked to draw the "reform student" I would paint a being that looks like an outer-space visitor, with a big head, probably a little heart, and a tiny chunk of body. That being would be mainly alone and mostly talk about mathematics learning, and would see the world through his school mathematical experience. That would be a "schizo-being" since she has a clearly divided self – one that has to do with mathematics and the other that has to do with other unrelated things. (p. 40)

From these perspectives, it is clear that the child in the mathematics classroom is 'socially constructed' as a predominantly cognitive, moral, future adult citizen of a democratic state and the mathematics that she is exposed to is an alchemic version of disciplinary mathematics, one that is more amenable to psychological and social "inscriptions". Quite clearly, this is an example of a fiction or a fabrication, not at all an inevitable description of the student in the mathematics classroom.

In conclusion, I believe that considering the discipline of mathematics as socially constructed has some implications for the teaching of mathematics in schools. In other words, if mathematics is not looked at as a fixed, eternal, and disembodied collection of truths and methods, but rather as a socially situated and historically contingent cultural practice, there is a possibility that a libratory, humanistic approach to mathematics, one that is aware of the historical and social forces that have shaped the discipline, could help to negotiate the terrain in the classroom without a sense of foreboding. After all, if the gods have been unmasked and shown to be all too human, there is a possibility that the fear and loathing that surrounds this discipline could be mitigated and all students, not just the privileged, can partake of its offerings.

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