Interplay between School Mathematics and Work Place Mathematics

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Sociocultural dimensions of mathematical knowledge have greatly influenced research in the field of mathematics education in the past few decades, resulting in the rise of different areas of research that include ethnomathematics, everyday mathematics, situated cognition, and workplace mathematics. The line of research on everyday mathematics has pointed out the importance of situations that evoke superior performance in quantitative reasoning in everyday settings and researchers have called for further investigation of everyday practices that involve mental mathematics. The general aim of this study is to develop a better understanding of the mathematics used in the workplace of bus conductors in Chennai, India. In particular, this study focuses on investigating the mental mathematics involved in bus conductors' work.

Introduction

Mathematical thinking and learning take place in any culture and it is important to study the mathematics of different groups of people from all over the world (Ascher, 2002). The mathematics education research community can benefit from such studies because such investigations broaden and modify the history of mathematics to one that has a multicultural, global perspective, and emphasize the interplay between mathematics and culture (ibid.). In the last two decades several researchers have analysed and documented the mathematics practices of adults as well as children, which take place outside the school settings (e.g., Carraher, Carraher, & Schliemann, 1987; Gerdes, 1996; Saxe, 1991). This association has given rise to the recognition of different forms of mathematics such as academic mathematics, everyday mathematics, folk mathematics, and ethnomathematics.

Ethnomathematics, Everyday Mathematics, and Workplace Mathematics

Ubiratan D'Ambrosio launched his *ethnomathematical program* in the 1970s "as a methodology to track and analyse the processes of generation, transmission, diffusion and institutionalisation of mathematical knowledge in diverse cultural systems" (D'Ambrosio, 1990, p. 78). One area of research that emanates from the research field of ethnomathematics is the research area of everyday cognition (Vithal & Skovsmose, 1997). One subset of the research field of everyday cognition is concerned with research conducted in workplaces. This line of research gives some insight into how people conceptualise the role of mathematics in their work. More recent research in workplace mathematics has attempted to uncover the mathematical practices of specific groups (e.g., nurses, automobile workers, carpet layers) and identify and establish abstractions that underlie these practices. Such studies assume significance because they "address lacunae in literature on the history of mathematical thought through the ages" (Presmeg, 2007, p. 449).

For several years now, key national educational policies of many countries (e.g., Australia, United States, and India) have suggested the need for integrating mathematics, work, and school (NCTM, 1989; National Council of Educational Research and Training, 2005). This integration does not mean narrowing mathematics education for serving short-term job-specific demands. Nor does it mean to prepare and train students for specific occupations. Instead the primary purpose here is "to make mathematics more authentic, thereby helping students to master knowledge and skills that are applicable to a wide range of situations"

(Hoachlander, 1997, p. 125). In order to address such issues, further investigation of workplace mathematics is needed as it would demonstrate "the unique forms of mathematics developed in workplaces and their synergies with formal school mathematics" (Zevenbergen, 2000, p. 185).

Although over the past 15 years, mathematics education research has begun to explore the nature of the mathematics used in different workplaces, very few studies have investigated the nature of workplace mathematics in India. Guided by the desire to add to the mathematics education research in India, the general aim of this study is to develop a better understanding of the mathematics used in everyday situations.

The line of research on everyday mathematics has found that people compute more efficiently mentally when problems are embedded in a certain context than when posed as a plain computational problem, and they have called for investigation of everyday practices that involve mental mathematics (Nunes, Schliemann, & Carraher, 1993; Saxe, 1991). This study will also contribute to the line of research pertaining to mental computation by investigating the mental mathematics involved in the bus conductors' work.

The research reported in this paper is part of a larger project that investigated the workplace mathematical practices of bus conductors in Chennai, India. The research purpose associated with this study is to observe, understand, describe, and analyse the mental mathematical practices of the bus conductors in their workplace. In this paper, I specifically examine one bus conductor's mental mathematical activities with a focus on bringing out the relationship between formal school-taught mathematics and workplace mathematics. I also discuss what this knowledge can add to the study of everyday mathematics and present educational implications.

Theoretical Framework

The assumptions underlying the present study are that the conductors' workplace mathematical practices are influenced by their working conditions and that their practice-linked goals emerge and change as individual conductors participate in this practice. Hence, I needed a theoretical framework that acknowledged the influence of context on their mathematical practices. Second, I needed a framework that would take into consideration the complex relationships between the conductors' mathematical tasks and the practices in which they were performed.

The three component analytical framework developed by Saxe (1991) is used to explore the research purposes of this study. The basic idea underlying his theory is that understanding of the mathematical environment needs the coordination of two perspectives – constructivist and sociocultural perspectives (Saxe & Bermudez, 1996). Saxe's framework consists of three analytic components that are concerned with goals, forms and functions, and the interplay among various cognitive forms. The first component pertains to the analysis of practice-linked goals that emerge and keep changing as individuals participate in their cultural activities, whatever they might be. The second component analyses how "cultural forms" influence the practice of the participants and how these forms shift in their functions with increased participation. His third component focuses on the interplay between cognitive processes of individuals who participate in distinct practices.

I use the third component of Saxe's (1991) framework to explore if the bus conductors used cognitive forms appropriated and specialized in the academic setting to their workplace settings. In particular, I investigated ways in which formal school-taught mathematics influenced bus conductors' mental mathematical activities.

Methodology

Research Methods and Data Collection

An instrumental case study approach was used to investigate bus conductors' mathematical practices. The bus conductors are employees of the government organization, Metropolitan Transport Corporation (MTC). Due to the hierarchical structures of the MTC, I first contacted the higher authorities of the MTC to gain permission to carry out my research with an employee (bus conductor) of the MTC. After gaining entry into the organization, a convenience sampling was done to choose a bus depot for investigation. A purposive sampling was employed and five participants were carefully and appropriately chosen based on participants' years of service, educational qualifications, service records, and their willingness to participate in the study. In this paper, I analyse one bus conductor's (Mrs. Radha) use of school mathematics to solve problems at work.

Mrs. Radha has been an employee with the MTC since 1980. In the year 1980, the government of Tamilnadu proposed a special ordinance that allowed women to apply for the post of bus conductors. Mrs. Radha was one of the few women who were selected for the job. According to her superior officers, she was extremely proficient in her job and she earned good collection everyday. She was born and brought up in the city of Chennai and completed all her schooling in government girls schools in Chennai. Her highest educational qualification is a pass in SSLC (equivalent to 10th grade). Mrs. Radha called herself an "excellent" student of mathematics. In primary school, she remembered doing mostly numerical problems that dealt with addition, subtraction, multiplication and division. She said that she really enjoyed learning the addition and multiplication tables and used them to do mathematical problems. She recalled doing some algebra and geometry in her high school mathematics classes. She said that because of her inability to continue her education, she lost touch with everything she learned back in high school.

During her work shift, Mrs. Radha commuted several times on a bus from point A to point B. She picked up and dropped off commuters en route and regulated their entry into and exit from the bus. Her duties as a bus conductor included issuing a ticket to a commuter based on the entry and exit point, tendering the exact change back to the commuter when the commuter gave more money than the required amount, keeping a record of the number of tickets sold, calculating the daily allowance based on the day's collection, and submitting the trip earnings to the supervisor at the end of his shift.

I accompanied and observed Mrs. Radha during her work shift three times a week, observing a total of ten trips. Based on the first few observations, I singled out factors that were helpful in pursuing the research question. Field notes and personal reflections were reviewed at the end of each day and participant's actions and events that needed further scrutiny were identified. Further, I collected and researched several official documents related to the research purposes associated with the study.

In order to examine the influence of formal school taught mathematics on Mrs. Radha's work related mental computational activities, I observed and identified several episodes from her work-related activities. During the follow-up interviews, stimulated recall technique was used to kindle her memory of these episodes. Mrs. Radha was probed to think aloud about the mental computational strategies she employed to solve work-related problems.

Analysis and Results

Based on my onsite observations, I identified situations where Mrs. Radha performed mental computation. Mrs. Radha engaged in mental computational activities when (1) she issued tickets to passengers during her bus trips, (2) when she determined overall earnings on her shift, and (3) when she calculated her daily allowance based on overall earnings on her shift. In order to understand if and how Mrs. Radha used school-taught mathematics at work, I used snippets from my observations of her work related activities and used stimulated recall technique to probe Mrs. Radha about her use of school taught mathematics.

Ticket Transactions

In order to give readers a glimpse of Mrs. Radha's mental computational activities, I present below a snippet from one of Mrs. Radha's several work-related activities during her bus trips (fieldnote).

> A passenger boarded the bus at point P along with three family members. Mrs. Radha approached the passenger. The passenger handed out a 50-rupee note and requested the conductor for four tickets to point Q. Mrs. Radha issued four tickets to the passenger and handed out thirty six rupees back to the passenger.

On this occassion, Mrs. Radha first determined the ticket fare for one passenger based on the entry point and the exit point. Next she calculated the total ticket fare for all four passengers and the balance amount due to the passenger. She handed out the money back to the passenger along with the tickets and moved on to the next passenger. This episode lasted less than 15 seconds. While Mrs. Radha described the goal associated with this event as "I issued tickets to a passenger", I described the mathematical activity associated with this event as "Mrs. Radha performed a mental mathematics transaction". In particular, my interpretation is that she solved the following computational problem mentally: $50.00 – (3.50 \times 4)$. I asked Mrs. Radha if she agreed with my interpretation and asked her to explain how she solved this computational problem.

Mrs. Radha: I agree that it is appropriate for the above situation. But I did not think of this problem as you have presented.

Researcher: Tell me what you did.

Mrs.Radha: I thought about it in a slightly different way. When I listen to passengers, I first determine the total ticket fare in terms of rupee and paise. I always separate the rupee part and the paise part when I do calculations. In the above situation, I first calculated 3×4 and got 12. I then knew I had to add four 50 paise (two rupees) to this total. So the total ticket fare is Rs. 14.00. Now your problem says that I have to subtract this amount from Rs. 50.00. I think I did that. But not the way you normally would do in school. I first gave Rs. 6.00 back to the passenger. Now I have accounted for Rs. 20.00 out of Rs. 50.00. I then gave another Rs. 30.00 to the passenger to get to Rs. 50.00.

Researcher: So you did not use any school taught mathematics at all?

Mrs. Radha: I do use it. In fact, I have used arithmetic facts a lot. However, I am using money mathematics side by side with those arithmetic facts instead of using school mathematics. In school we use long multiplication to calculate 3.50 × 4. If I use that technique at work, it will take a lot of time. If I use my technique mentioned earlier, I could do very quick mental calculations.

Researcher: So what kind of mathematics are you using?

Mrs. Radha: I use arithmetic facts and the concept of money. But I combine them both and come up with an easier method to solve problems at work. To complete my duties efficiently, I use this type of maths. It is a combination of school maths and money maths. You can call it conductors' maths.

It is clear from the above explanation that the use of monetary units in the mental transactions helped Mrs. Radha complete ticket transactions correctly, quickly, and efficiently.

Calculation of Overall Earnings

Every day at the end of her shift, Mrs. Radha determined overall earnings from her shift. She used an official record called a *waybill* to record the number of tickets sold in each denomination and used this information to calculate overall earnings. In table 1, I present information adapted from Mrs. Radha's waybill contents (official record).

Ticket denomination	Total number of tickets sold	Total amount collected
2.00	424	848.00
3.00	349	1047.00
3.50	123	430.50
4.00	79	316.00
4.50	43	193.50
Token ticket	1	30.00

Table 1. Total earnings by ticket denomination

I observed Mrs. Radha as she calculated the total amount corresponding to each ticket denomination. Mrs. Radha referred to the 3.50 ticket denomination and said that normally at school she would solve 3.50 × 123 using long multiplication method. But at work, she said that she resorted to conductors' maths to determine ticket collection amounts. She explained the way in which she mentally calculated the total ticket earnings for each denomination.

> I have sold 123 tickets corresponding to 3.50 ticket denomination. I don't use school method because it is time consuming and hard to do mentally. Instead our (conductors') technique is to calculate ticket amount for every 100 tickets. Thus the collection amount is Rs. 350 for the first 100 tickets. I should now account for the remaining 23 tickets. I first calculate ticket amount for the first 10 tickets, which is Rs. 35.00. I double it to get Rs. 70.00 for 20 tickets. I now add Rs. 350 and Rs. 70 to get Rs. 420. I then calculate collection amount for 3 more tickets, which is Rs. 10.50. Now I add this to Rs. 420 to get Rs. 430.50. I do it all in my mind very quickly using this method.

Using the above technique, Mrs. Radha was able to do quickly determine daily earnings using data from the waybill. Further, she said that she successfully used this form of mathematics to work on new bus routes with varying ticket denominations.

Calculation of Daily Wages

Mrs. Radha receives a daily allowance for every shift she worked on. This amount depends on the overall collection she makes on her shift. She calculates this allowance amount at the end of her shift and shares this amount equally with the driver of her bus. The daily allowance is comprised of fixed allowance (Rs. 18.50) and variable allowance. For every 100 rupees collected, the variable allowance is fixed at Rs. 2.35.

I observed and listened as Mrs. Radha explained her mode of calculating the daily allowance corresponding to data presented in table 1. The total collection amount presented in table 1 is Rs. 2865. Mrs. Radha said that instead of calculating 2865 × 2.35 using regular methods (school-taught), she used a different approach to determine the allowance amount. I summarize Mrs. Radha's methods in table 2.

Mrs. Radha: I split 2865 into 2000, 500, 300, 65. I calculated the variable allowance corresponding to these amounts. Variable allowance is fixed as Rs. 2.35 for every 100 rupees collected. Thus for every 1000 rupees the allowance is Rs. 23.50.

Collection amount	Allowance (In rupees)	Method
2000	47.00	Double 23.50
500	12.00	Round off 23.50 to 24 and divide by 2.
300	7.00	Triple 2.35 to get 7.05; ignore 5 paise
65	1.20	Approximately half of 2.35
2865 (total)	67.20	Sum of allowances

Table 2. Variable allowance calculation

Mrs. Radha determined the overall allowance by adding the fixed allowance (Rs. 18.50) to her variable allowance (Rs. 67.20). I asked Mrs. Radha about her approximations in steps two, three, and four. She said that since the final allowance (Rs. 85.70) is always rounded off to the nearest rupee (Rs. 86.00), such minor approximations would not result in major discrepancies in her allowance amount.

Using exact calculations, the overall allowance (in rupees) is determined as follows:

$$2.35 \times 2865 = 67.30; \; 67.30 + 18.50 = 85.80$$

This amount (Rs. 85.80) when rounded to the nearest rupee gives the allowance amount as 86.00, which Mrs. Radha arrived at using her methods. I summarise Mrs. Radha's use of school mathematics at work using her own words below.

> I think I use lot of ideas from school mathematics at work. However, it would be very hard to use only school taught algorithms at work. I have to use parts of what I learned at school and then adapt it and use it in conjunction with other forms (use of money maths) to be an efficient conductor.

Discussion

It has been documented elsewhere that bus conductors' workplace mathematics has certain unique characteristics that are shaped by the context and the tools specific to their workplace. In fact, I argue that conductors' workplace mathematics is the result of integration of work-specific knowledge, formal school-learnt mathematical ideas, and the knowledge about the currency system (Naresh, 2008).

Focusing our attention to the present discussion, Mrs. Radha's workplace setting demanded that she perform mental mathematical calculations quickly and efficiently. In the absence of technological devices, she had to work carefully to avoid errors in calculations. The results of the data analysis suggest that Mrs. Radha acquired, adapted, and used components of school mathematics to solve computational problems related to work successfully. Specifically, she appropriated and adapted ideas related to arithmetic that she had developed and specialized in the school setting to solve mental mathematical problems in another setting – at work. Mrs. Radha's use of conductors' maths at work shows evidence of interplay between cognitive forms that Mrs. Radha developed at school and at work.

Educational Implications

Although there are several contexts in which mathematical ideas are discussed, mathematics education is mostly associated with the institutional context (Pinxten, 1994). The problem is that usually in the school setting, mathematical knowledge is decontextualized, which leads some students to be alienated from the learning of mathematics (Guberman, 1996). One possible solution to overcome this problem is to bring components of everyday mathematics into classrooms. One possible way to bring components of everyday mathematics into classrooms is to use mathematical tasks that closely resonate with children's everyday activities (Freudenthal, 1991; NCTM, 1989).

Children are introduced to the concept of money at a young age. This topic, which is considered "a bridge between everyday life and school," could be used to provide an authentic context to design problems relevant to everyday activities (Brenner, 1998, p. 150). Bus conductors' work and work-related activities could serve as a source for designing such problems to integrate students' learning experiences in and out of school. Further, the integration of mathematics, work, and school could provide opportunities for students to learn mathematics in the process of searching for solutions to meaningful work-related problems.

The findings from this study and other such studies suggest that adults seldom use traditionally taught mental computation and estimation strategies to solve problems outside school settings. Despite the fact that 80 percent of adult usage of mathematical activities involves estimation and mental computation, 80 percent of the time in elementary school instruction is spent on teaching written algorithms (Reys & Nohda, 1994). Another implication that could be derived from this study specifically concerns the use of contextual tasks to introduce and teach mental computation and estimation in elementary school classrooms. By using tasks set in the context of money teachers could make mental computation meaningful to students and could evoke superior performance from students in quantitative reasoning situations.

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