Rote and Algorithmic Techniques in Primary Level Mathematics Teaching in the Light of Gagne's Hierarchy

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The rote and algorithmic methods based on 'Sutras' in traditional Indian schools are looked down upon by the majority of modern educationists. This paper examines these techniques in the light of Robert Gagne's theory of hierarchy of concepts and meaning. A semantic net model has been presented and the techniques of rote and algorithmic learning as well as the principle of constructivism are examined in its light. It is shown that the traditional techniques prepare the foundation necessary for concept formation. On the basis of this, it is suggested that teaching method be developed that properly utilizes the insights on cognition provided by Gagne.

Introduction

Rote and algorithmic methods are being used in traditional primary schools of India for teaching Language and Mathematics since a long time. This system of teaching has been prevalent since as far back as 200 BC and is still continuing up to the present time (Altekar,1992). The term 'algorithm', in fact, is derived from the name of the Arab mathematician *Al Khwarizmi* who had visited India around 9th century AD and introduced Indian mathematics to the western world on returning back through various textbooks that he wrote such as *Al Jabar* (from which the term Algebra has come), *Muqabla-e-Hindsa,* etc. The term 'algorithm' is mainly used today in computer science and refers to a step-by-step breakup of the procedure for a given computational task. A number of popular computational algorithms had been developed in ancient India which are still taught in the Indian traditional schools. The students are made to practice rigorously the various computational procedures. This makes them quite proficient in day-to-day computations. Rote learning is an important aspect of the Indian teaching method for mathematics. It is one of the goals of this paper to discuss the significance of rote as well as algorithmic methods of teaching as a preparation for concept formation.

A second important feature of the Indian method for mathematics teaching is the use of *Sutra* for imparting teaching. Ancient Indian scholars had amassed a vast amount of knowledge in different fields and organized them in *Sutra* form (Jaiswal, 1997, p4). *Sutra* (Sanskrit) has been defined in Amarkosha as that by which fabric is woven (Oak, 1913). The *Sutra* form, as we shall see, is a semantic net representation that had been designed for oral transmission of knowledge and oral teaching that was prevalent in ancient India. Today we are again starting to explore semantic net-based pedagogical tools such as concept maps (Novak, 1990).

Many modern educationists are of the view that rote learning without understanding should have no place in education. They look down upon the traditional rote-based teaching methods (e.g. National Curriculum Framework, 2005, sect.1.4). So in the school-curricula the rote-based teaching is gradually being replaced by newer methods based on heuristics and constructivism. Heuristic methods are discovery based (Polya, 1957, p. 112). In constructivism the child is considered as a constructor of knowledge (National Curriculum Framework, 2000, p. 26). Children do not exactly learn what we teach them. Rather, they pick and choose from their environment to build up their own concepts. The constructivist teacher acts as a facilitator for the child. The modern viewpoint regarding rote learning is reflected in the following excerpts from Gagne (Gagne, 1970, p. 100), "Although few educators would be inclined to say memorization is bad, it is generally thought to be unimportant. Perhaps it is a matter of priority. Students must acquire knowledge and the ability to think, and these goals are heavily emphasized in today's curricula.". Ironically, it is just this sense of urgency to give the students knowledge and thinking ability that is denying the students the abilities that they desire. Let us see how.

A large number of students today are extremely poor in their basic concepts (Agnihotri et al., 1994, pp. 86-87). One reason for this weakness in concept-based subjects is that concept formation relies on basic language and mathematical skills which in the initial stages, are helped by rote learning. If the proficiency in language and mathematics is poor, it leads to a poor comprehension of text as well as mathematical statements. The present-day curriculum-builders have undermined the importance of rote learning as a tool for gaining proficiency in language and basic mathematics. The students of today do not memorize multiplication tables and vocabulary as much as their elders used to do. These aspects are getting neglected when knowledge-based subjects such as social studies and science are introduced in lower classes where the student is inadequately prepared to handle them. For example, it is typical in Indian schools teaching in the English medium, to give word problems right from class 1. Elements of set theory and number line concept are introduced in the kindergarten stage. The students struggle and waste a lot of time and effort. Concepts such as force and work are taught again and again starting from Class 4. The treatment is always partial because the student has not learned the necessary tools. Some experts feel that rote learning makes the mind dull. But stressing the student's mind beyond their comprehension ability as is done today forces the student towards those aspects of rote memory that the modern educationists are trying to avoid. Methods based on heuristic and constructivist approaches have been tried out in different schools at various levels on an experimental basis. There have been some remarkable successes where dedicated teachers were involved (Agnihotri et al., 1994, pp. 137-146). But attempts to generalize them have not been effective. What is so bad about rote methods?

Here I would like to quote another remark of Gagne (Gagne, 1970, p. 101).

> "It is no accident that some of the most renowned American orators of previous generations knew such literature as the Bible and the writings of Shakespeare to such extent that they could repeat the passages verbatim. ... People who are skilled in oral communication are able to recall words, phrases or entire passages of flowing English, and weave them in their own vocal utterances in highly effective ways. They are able to do this not simply because they have read these classics of English literature, but because they have *memorized* them. In the terms used … they have learnt many varieties of verbal sequences, and they can recall them readily."

Thus rote learning does have its virtues. Today instructional techniques based on constructivism as well as heuristic approaches are being advocated. But for some reason or other, the expected success has not been obtained. We will have to understand the cognitive mechanism through which mental reconstruction and heuristics work. The present paper focuses on some aspects of the mechanism of concept formation and shows that if constructivist as well as heuristic approaches are to succeed, then they should fulfil certain pre-requisites. The pre-requisites can be comprehended in terms of Robert Gagne's taxonomy that he has described in his book 'The Conditions of Learning' (Gagne, 1970). I would like to show that these pre-requisites are effectively provided in the various tools of traditional Indian education.

Gagne's Hierarchy of Concept and Meaning

Robert Gagne in his book 'On the conditions of Learning', has given a taxonomy of learning types (Gagne, 1970 Chap.4). that he has arranged hierarchically.

1. *Signal learning* This is a type of associative learning that has been initially studied by Pavlov who has called it conditioned reflex. A subject that responds in a certain way (R) to a stimulus S_1 is given two stimuli (S_1 and S_2) simultaneously. After sufficient number of repetitions he learns to give the response (R) to S_2 even in the absence of S_1. Much of the learning that we do without giving conscious thought is of this type. Much of the initial learning of early childhood is signal learning.

2. *Stimulus-response learning* This is another type of associative learning that has been called trial and error learning by Thorndike. Skinner has used the term operant learning for it. It involves some goal or objective that the subject attempts to achieve. The process is essentially a successive approximation process. The initial efforts are almost random. The subject modifies his approach in every attempt. Each successful attempt is remembered while failed attempts are forgotten. The success rate improves with more attempts. A good example is a child learning to walk. Initially he falls down often. But with more attempts he is able to master the skill.

3. *Chaining* Chaining is the process of establishing a sequential connection of a set of stimulus-response pairs for the purpose of attaining a particular goal. For example, the opening of a lock involves a number of simpler steps connected in a sequence (locate the key-hole -

insert the key - turn the key clockwise - watch for lever unlocking - take off the lock). Successful chaining requires prior learning of each component response. Algorithms are generally such chaining sequences.

4. *Verbal Association* Human beings have the ability to encode and express knowledge through sound patterns. Verbal association here refers to the most elementary kind of verbal behavior - learning of verbal associations (object « name) and verbal sequences (chains of verbal associations).

5. *Multiple Discrimination Learning* discrimination is the ability to distinguish between two or more stimulus objects or events. There are two different kinds of capabilities involved. The first is where the learner is able to make different responses to different members of a collection of stimulus events and objects. The second type involves the capability of the learner to respond in a single way to a collection of stimuli belonging to a single set. (This involves recognition of the defining rule for the set and responding accordingly.)

6. *Concept learning* Concept learning involves discrimination and classification of objects**.** We will distinguish between two types of concept learning: concrete and abstract. Concrete concepts are those that are formed through direct observation. For example, consider the edge of a table, the edge of a razor blade and the edge of a cliff. It is possible to formulate a rule that defines an edge. But the concept of edge is formed more easily through direct observation of several examples. A learner can respond to a set of stimulus objects in two ways – one by distinguishing among them and the other by putting them into a class and responding to any instance of that class in the same way. Both these types are examples of concept learning. The significance of concept learning is that it frees the learner from the control by specific stimuli.

7. *Principle (or rule) learning* Some concepts are not concrete. They are based on rules that involve other concepts. So they have to be learnt through definition. Definitions are statements that express rules for classifying, i.e. rules that are applicable to any instance of a particular class. Definitions are used for objects as well as for relations. A salient feature of principle learning is that the learner cannot acquire the concept through memorizing its statements verbatim unless he knows the referential meanings of the component concepts. For example, $2H_2 + O_2 = 2H_2O$ is meaningless unless you understand what the symbols H_2, O_2, and H_2O represent and are familiar with the mole concept. The concept formation process is cumulative. It weaves the different objects into a semantic web. Such a semantic web has been described by Novak, for example, as a concept chart (Novak, 1990) and by Gagne as a learning hierarchy (Gagne, 1970, p. 142).

8. *Problem solving* Problem solving, here, refers to something more than classroom mathematical drills. Also referred to as heuristics (Polya, 1957), the process of problem solving is one in which the learner discovers a combination of previously learnt rules that can be applied to achieve a solution for a novel situation. The following sequence of events is typically involved in problem solving. (1) presentation of the problem, (2) definition of the problem, (3) formulation of hypothesis, (4) verification of hypothesis. The learning outcome of problem solving is a higher order rule that becomes a part of the student's repertory.

According to Gagne, cognition and concept formation is a multi-layered phenomenon, each layer consisting of a particular learning type. Signal learning, Stimulus-response learning, Chaining, Verbal Association and Multiple Discrimination Learning are all pre-requisites for the formation of concepts and the ability to solve problems. The process of concept formation involves all these eight processes. A very important point here is that if the learning has not been sufficiently accomplished at any level, then there is perceptible deterioration at all higher levels (Gagne and Wigand, 1970).

Rote Learning in the Context of Gagne's Hierarchy

Let us examine the different hierarchy levels of Gagne and see where the traditional methods of teaching fit. Signal learning, Stimulus-response learning, Chaining, Verbal Association and Multiple Discrimination Learning constitute the basic forms of learning. They are the basic building tools that enable the mind to acquire a working set-up for concept formation. It is this area where rote learning is most effective and insufficient learning at this level impairs the student's abilities for higher learning.

Signal learning refers to learning through unconditional association. When small children memorize alphabets and digit symbols, they are unconditionally associating the symbol sounds with their form. Since the child does not as yet possess any related pre-formed associations, this is the only learning alternative available at this stage. Rote learning is the most effective learning tool at this stage because it directly does what is required.

Stimulus-response learning or Operand Conditioning is a process based on successive approximation. Once the basic nodal associations have been formed in the mind, a successive approximation process or shaping takes place on the basis of positive and negative reinforcements. In the traditional elementary education, this step is accomplished through a lot of oral exercises.

The next step, Chaining, is the process of combining a set of individual S-R's in sequence. In fact, the concept of *Sutra* developed in ancient India (Namita, 1996), is a formalisation of this step. *Sutra* has been identified with algorithm by Vernekar. The term 'algorithm' refers to a step-by-step method for solving any problem (Rajaraman, 1980, p.3). According to Vernekar (1994), the basic idea of the algorithmic method is that the various steps in an operation are arranged like beads in a thread (sutra). Thus sutra as well as algorithms refer to the same process as chaining or forming mental links.

The next step is verbal association. Most of the beginners' verbal associations are definitions and fact-snippets to be memorized. Here again, the rote methods are applicable. Although memorizing the vocabulary is a very boring job, once a student acquires good vocabulary through whatsoever means, its role in understanding verbal and written material cannot be denied.

How the Present-day Mathematics Teaching Violates Gagne's Principle

Present day curricula stress the role and necessity of concept formation in education (National Curriculum Framework 2000, 2005). This cognitive approach appears to be quite reasonable. A cognitive approach can be very useful in this context (Redish, 1994). At present, the heuristic-constructivist approach is being implemented in the modern schools for teaching mathematics as well as other science subjects.

As I have discussed earlier, a majority of students who go to higher classes are found to be extremely poor in concepts (Agnihotri et al., 1994). Arons (1997) has pointed out several deep conceptual flaws in the thinking of average Physics students. Why do conceptual flaws occur? Assuming Gagne's model, the following learning types heavily rely on previously learned materials. (1) Chaining, (2) Multiple discrimination learning, (3) Concept learning, (4) Principle learning, and (5) Problem solving. And as we have seen, the kind of learning material that these learning types are based on is most effectively done through memorization.

In mathematics, one who has memorized the multiplication tables and rigorously practiced basic mathematical operations through oral methods is much more confident in higher mathematics because he has less stumbling blocks to overcome.

Modern school education has gradually done away with basic mathematical drills. So the pre-requisites for formation of higher concepts as pointed out by Gagne are not being fulfilled. Mathematical knowledge is cumulative in nature. So with a weak foundation the majority of students are bound to display an overall weakness in their concepts. This, according to my view, is the main reason why many of today's students are weak in concepts.

Gagne's Hierarchy and Semantic Net Representation

Semantic net is a useful way for representation and organization of knowledge in the mind. Basically, a semantic net is a representation - a set of conventions for describing a class of mental objects. Margaret W. Matlin (1995, p.217) has given an excellent description of several semantic net implementations. These models use graph theory concepts for representing knowledge-bases. I will present here a simple semantic net model and show that it incorporates both Gagne's Hierarchy as well as the principles of constructivism.

Semantic Net Elements

The model that I will consider is based on the following abstract semantic net elements: Consider a simple sentence. *Ram eats a ripe mango*. We can easily identify the following elements:

1. *Object or entity:* The words *Ram* and *Mango* are representing objects or entities.
2. *Attributes of a particular entity:* The word *ripe* is representing an attribute of *Mango.*
3. *Relation between entities:* The phrase *is eating* is representing a relation between *Ram* and *Mango*.
4. *Layer:* It consists of network of objects and relations. This sentence might be part of a story or some event description. Such a passage representing a complex situation can be represented as a network of [object-relation-object] and [object-attribute] elements.
5. *Knowledge-base* consists of one or more such layers.

A knowledge-base is represented as a simple or complex network of several inter-related elements. These elements are called objects or entities. Each entity has its own set of attributes or properties that can be used for the purpose of association or classification. Two objects can be associated through a relation. The relation acts as a link between peer objects (objects in one layer). The link connects objects semantically. Several objects can exist in a conceptual layer. The objects in a given layer when interlinked form a network. A knowledge-base contains several layers. Sometimes they are well-defined. Sometimes they are not. Sometimes some links are missing.

The process of concept formation can be viewed as formation of a corresponding semantic net in the mind. The semantic net model assumes that well-formed concepts are based on well-formed representations of the corresponding knowledge-base in the mind.

Semantic Net Layers and Gagne's Hierarchy

There is a striking correspondence between Gagne's classification and the semantic net representation. The eight learning types in Gagne's hierarchy can be divided into two categories. The first five types consisting of signal learning, stimulus-response learning, chaining, verbal association and multiple discrimination learning can be viewed as related to the learning of the three lower-level components in the semantic net representation (Object or entity, Attributes of a particular entity, and Relations between entities). The upper three layers consisting of concept learning, principle learning and problem-solving will then represent processes involving larger chunks of knowledge-base, i.e. related to layers and sets of layers.

How do we practically understand it? When you start with a new field of knowledge of which you do not have any previous experience, learning is very difficult because you have nothing to associate any new idea with. The first few key ideas must be established as nodes in the mind through signal learning techniques. The child has to memorize the alphabets and basic mathematical operations. At this stage, concept has no meaning. This process corresponds to fixing up the single **entities** in the mind so that they can be recalled whenever needed. This is accomplished through rote learning. Thus the rote learning stage is important because until a skeleton network is established, nothing can be added to it as further association. And establishing skeleton network through any other method is not quite practical.

Stimulus-response learning as well as verbal associations are possible only after the lowest level network has been established through signal learning. Through this process the network is expanded and refined as newer situations are encountered. The **entity-attribute** associations are mainly shaped during this stage so that entities can be better recognized and discriminated.

Then only through chaining, the various **entity-relation-entity** links are created. This stage is possible only after one has gone through the second stage so that entities are properly recognized and discriminated. The third level continues until the network is complete. Verbal association takes place throughout this process.

Until the above stages are more-or-less completed, it is very difficult to smoothly carry out the three upper layer operations. For example, a number of textbooks today that have been written with heuristic learning pattern in mind put up a lot of questions without answering them. But without adequate information content, it just puzzles the mind more and more. It is a blunder to think that an average student will discover the missing links of a knowledge base without access to the required information.

The upper three layers consisting of concept learning, principle learning and problem-solving are operations related to the multi-layer knowledge-base as a whole. Any layer of concept is based on well-learnt hierarchically lower-level concepts. The transmission and reconstruction of a particular layer of concept is successful only when the semantic nets of the lower levels are well-formed in the mind.

Communication of Knowledge from Semantic Net Viewpoint

Communication is the process of sharing mental images, abstract concepts and parts of knowledge bases between different minds. Strictly speaking, these cannot be shared. But an approximate copy of the mental image of one person can be transmitted to another person. Language – verbal, symbolic or pictorial, becomes the medium for this. Language elements are used for encoding a particular semantic net for transmission. The teaching-learning process consists of isolating small segments of related objects for the purpose of communication and converting them to a language-based format before transmission in signal form through sensory-motor channels. These signals are received by the recipient which is followed by reconstruction of a corresponding mental picture or knowledge base in the recipient's mind in his own format.

Sutra as a Representation for Semantic Net Elements

In ancient India, *Sutra* was developed as a tool for oral as well as written knowledge transmission. A good *Sutra* has been defined as one with the following attributes (Choudhary, 1997, pp. 6-7): It is (1) expressed with a very small number of letters, (2) unambiguous and beyond doubt, (3) containing the essence of the respective field of knowledge, (4) balanced from all viewpoints, (5) free from internal contradictions, (6) error-free. Thus a *sutra* contains essential or important knowledge represented in a compact and unambiguous format. Following types of *sutra* have been composed. (1) *Sangya sutra* for naming (Node definition). (2) *Paribhasha* or Definition (Node or link definition), (3) *Vidhi sutra* or algorithm (Chaining of node-link-node combinations for specific purpose), (4) *Niyama* or Laws (logical propositions), (5) *Vishesha* or exceptions, and (6) *Atidesha* or analogy. We find that a *Sutra* is nothing but a compact representation in language form of a semantic net component. It consists of either a node-definition or node-attribute association or a node-link-node combination in the semantic net for a specific topic. It is knowledge in very compact form and requires proper exposition to bring out its full implication. Traditional Indian schools have been using Sutra-based teaching methods for mathematics teaching as well as other conceptual subjects. For this purpose they had composed rhythmic sutras that can be recited. Once memorized, they acted as a knowledge base that was very useful for problem-solving situations and for higher concept building.

Conclusion

I have attempted to show in this paper on the basis of Gagne's information processing model that the rote and algorithmic methods used in traditional Indian schools are effective in building a strong base for formation of higher concepts. I want to suggest that we should develop teaching methodology for mathematics and other subjects that incorporates rote learning in an effective way so that knowledge is better conveyed and represented in the mind of students. The rote learning of basic mathematical facts and word-meaning in primary schools will in particular be a very useful preparation for higher concepts. For better results a balance between heuristic approach and algorithmic approach will have to be established. We should also develop effective uses of sutra in mathematics teaching.

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