

Introducing Fractions Using Share and Measure Interpretations: A Report from Classroom Trials

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It is well acknowledged that 'fractions' is one of the most complex topics in the primary and middle school curriculum. There has been a considerable amount of research on teaching and learning of fractions in the last few decades and the curriculum design in the West. A Non-Governmental Organisation (NGO) in collaboration with an education research institute is engaged in similar attempts in the Indian context. This paper is a report of our trials in two schools on introducing fractions to primary school children using a combination of share and measure interpretation.

Introduction

It is well acknowledged that 'fractions' is one of the most complex topics in the primary and middle school curriculum. There has been a considerable amount of research on teaching and learning of fractions in the last few decades and the curriculum design in the West has been informed by such research (Kieren, 1993; Lamon, 1999; Mack, 1993). In India a typical school curriculum introduces fractions in grade 3 and by grade 5 a child is expected to have learnt to compare fractions and be able to perform all the operations on them. A quick look at some of the textbooks will reveal that after a brief introduction to fractions using the part whole subconstruct, the books take an algorithmic approach¹. As a result children learn to perform a series of operations mechanically without any conceptual grasp. School education in India suffers from a range of ills and curriculum research is still an emerging area in India. How-

ever there have been attempts by several groups (for example Jodogyan, Prashika of Eklavya, Digantar School) across the country to try and intervene and improve the quality of mathematics education and more often than not such attempts try to prioritise conceptual grasp over algorithmic learning. On the topic of fractions, these interventions attempt to simultaneously introduce a variety of fraction sub-constructs and expect students to solve word problems relating fractions to real life situations. However, feedback from classroom trials suggests that such approaches are still conceptually too dense for children at the primary school level to grasp, prompting some educators to suggest that we do away with teaching fractions at the primary school level and instead introduce decimal fractions². It is in this context that the groups working on curricular research in mathematics education at an NGO, Eklavya, and the Homi Bhabha Centre for Science Education (HBCSE) came together to attempt an alternative approach as part of the longer study – *Connecting the fraction sub-constructs for the development of students' understanding of ratio and proportion situations*. We used a combination of share and measure interpretation of fraction and though the two groups interacted and shared their experience, there is a difference in the emphasis placed on the two interpretations. We³ adopted an approach that placed a higher emphasis on share interpretation and brought in the measure interpretation at a later time. This paper presents our approach, its effectiveness and shortcomings.

¹ There are notable exceptions to this. For example the NCERT textbooks introduce fractions only from grade 4 and emphasizes conceptual understanding.

² Much of these are not documented and published. These were views expressed by some who have seriously engaged with teaching fractions, during the course of discussions on teaching fractions to school children.

³ 'We' from now on will refer to the NGO, Eklavya, that conducted this study.

Our Experience with Teaching Fractions

We have been actively exploring different approaches to teaching fractions to middle school children for the last three years. Our initial explorations were with grade 7 children who had a prior exposure to fractions and operations on them. In grade 6 they were supposed to have learnt ratio, proportion and percentages. The grade 7 syllabus moved over to 'Rational Numbers' where for the first time they will encounter fractions whose numerator or denominator could be a negative number and they will learn the different properties that make the set of rational numbers a 'field'. However, children had no meaningful association for the symbol $\frac{m}{n}$ or for operations on them. We intervened by introducing some activities like paper folding or shading squares and focused on conceptual grasp. We used a lot of worksheets and incorporated word problems that children could relate to. We encouraged the children to think and ask questions rather than practice what we taught them. Though all these seemed to make their engagement with fraction more meaningful and after instruction students usually performed better, a short gap of few days was enough to erase from their mind all that they learnt and they would slip back to committing the same kind of errors that they did before our intervention. Moreover, coming at a later stage in their learning, they found the activities quite boring and the better of them were more eager to demonstrate their skill at algorithms rather than to acquire a conceptual understanding.

We also conduct teacher-training programs and the training material that we developed for a week long training used the *measure interpretation* for defining a fraction and used the area model predominantly. However we ourselves could never take the training material to the classroom because it was conceptually loaded, introduced different subconstructs of fraction simultaneously and addressed a range of issues. It is at this stage that we started our interaction with the group at HBCSE acquired access to current literature on teaching and learning of fractions. Inspired by Streefland's work, this time we tried to introduce fractions using the share interpretation. The trials were conducted on two groups of 10 to 12 year old children, one (consisting of 15 children) at another NGO (Muskan) working with slum children in a city and the other (a total 17 children belonging to grades 5 and 6) at private school catering to children from lower income group in a small town. The groups had almost no prior introduction to fraction when the trials were conducted. The classes were spread over a period of three months with several gaps inbetween.

Share and Measure Interpretations of Fractions

In the *equal share interpretation* the fraction $\frac{m}{n}$ denotes one share when m identical things are shared equally among n . The relationships between fractions are arrived at by logical reasoning (Streefland, 1993). For example $\frac{1}{6}$ is the share of one child when 5 rotis (disk-shaped handmade bread) are shared equally among 6 children. The sharing itself can be done in more than one way and each of them gives us a relation between fractions. If we first distribute 3 rotis by dividing each into two equal pieces and giving each child one piece each child gets $\frac{1}{2}$ roti. Then the remaining 2 rotis can be distributed by dividing each into three equal pieces giving each child a $\frac{1}{3}$ piece. This gives us the relations

$$\frac{5}{6} = \frac{3}{6} + \frac{2}{6} \quad \text{and} \quad \frac{5}{6} = \frac{1}{2} + \frac{1}{3}$$

The relations $\frac{3}{6} = \frac{1}{2}$ and $\frac{2}{6} = \frac{1}{3}$ also follow from the process of distribution. Another way of distributing the rotis would be to divide the first roti into 6 equal pieces give one piece each to the 6 children and continue this process with each of the remaining 4 rotis. Each child gets a share of $\frac{1}{6}$ roti from each of the 5 rotis giving us the relation

$$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

It is important to note here that the fraction symbols on both sides of the equation have been arrived at simply by a repeated application of the share interpretation and not by appealing to prior notions one might have of these fraction symbols. In the share interpretation of fractions, unit fractions and improper fractions are not accorded a special place. Also converting an improper fraction to a mixed fraction becomes automatic. $\frac{6}{5}$ is the share that one child gets when 6 rotis are shared equally among 5 children and one does this by first distributing one roti to each child and then sharing the remaining 1 roti equally among 5 children giving us the relation

$$\frac{6}{5} = 1 + \frac{1}{5}$$

Share interpretation does not provide a direct method to answer the question 'how much is the given unknown quantity'. To say that the given unknown quantity is $\frac{3}{4}$ of the whole, one has figure out that four copies of the given quantity put together would make three wholes and hence is equal to one share when these three wholes are shared equally among 4. Share interpretation is also the quotient interpretation of fractions in the sense that $\frac{3}{4} = 3 \div 4$ and

this is important for developing students' ability to solve problems involving multiplicative and linear functional relations (Subramaniam & Naik, 2007).

Measure interpretation defines the *unit fraction* $\frac{1}{n}$ as the measure of one part when one whole is divided into n equal parts. The *composite fraction* $\frac{m}{n}$ is as the measure of m such $\frac{1}{n}$ parts. Thus $\frac{3}{5}$ is made of 3 piece units of size $\frac{1}{5}$ each and $\frac{2}{3}$ is made of 4 piece units of size $\frac{1}{3}$ each. Since 5 piece units of size $\frac{1}{5}$ make a whole, we get the relation $\frac{3}{5} = 1 + \frac{2}{5}$. Significance of measure interpretation lies in the fact that it gives a direct approach to answer the 'how much' question and the real task therefore is to figure out the appropriate n so that finitely many pieces of size $\frac{1}{n}$ will be equal to a given quantity. In a sense then, the measure interpretation already pushes one to think in terms of infinitesimal quantities. Measure interpretation is different from the part whole interpretation in the sense that for measure interpretation we fix a certain unit of measurement which is the whole and the unit fractions are sub-units of this whole. The unit of measurement could be, in principle, external to the object being measured.

Introducing Fractions Using Share and Measure Interpretations

One of the major difficulties a child faces with fractions is making sense of the symbol $\frac{m}{n}$ and what quantity it represents. In order to facilitate students' understanding of fractions, we need to use certain models. Typically we use the area model in both the measure and share interpretation and use a circle or a rectangle that can be partitioned into smaller pieces of equal size. Circular objects like roti that children eat every day have a more or less fixed size. Also since we divide the circle along the radius to make pieces, there is no scope for confusing a part with the whole. Therefore it is possible to avoid explicit mention of the whole when we use a circular model. Also, there is no need to address the issue that no matter how we divide the whole into n equal parts the parts will be equal. However, at least in the beginning we need to instruct children how to divide a circle into three or five equal parts and if we use the circular model for measure interpretation, we would need readymade teaching aids such as the circular fraction kit for repeated use. Rectangular objects (like cake) do not come in the same size and can be divided into n equal parts in more than one way. Therefore we need to address the issues (i) that the size of the whole should be fixed (ii) that all $\frac{1}{2}$'s are equal—something that children do not see readily. The advantage of rectangular objects is that we could use paper models and fold or cut them into equal parts in different ways and hence it easy to demonstrate for example that $\frac{3}{5} = \frac{6}{10}$ using the measure interpretation (See. Figure.1)

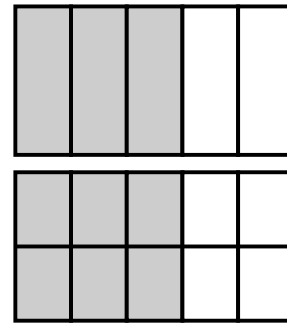


Fig. 1. Rectangular area model showing $\frac{3}{5} = \frac{6}{10}$

Though we expose children to the use of both circles and rectangles, from our experience we feel circular objects are more useful when use the share interpretation as children can draw as many small circles as they need and since the emphasis not so much on the size as in the share, it does not matter if the drawings are not exact. Similarly rectangular objects would be more suited for measure interpretation for, in some sense one has in mind activities such as measuring the length or area for which a student has to make repeated use of the unit scale or its subunits.

Our Study

Representing Fractions

We introduced fractions using the share interpretation. The fraction $\frac{3}{4}$ was defined as the share of each child when 3 identical things were equally shared among 4 children. The first few classes encouraged children to draw pictures and show, in how many different ways a set of identical things (usually rotis) can be shared equally among a set of children. They were also encouraged to write the share of each child resulting from the sharing exercise in the form of an equation. For example if three rotis were shared among four children by sharing one by one each roti equally among four children, they were required to write

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

If they did the sharing by first sharing two rotis between four children by dividing each roti in to two equal parts and then the remaining one roti among four children, they were required to write

$$\frac{3}{4} = \frac{2}{4} + \frac{1}{4} = \frac{1}{2} + \frac{1}{4}$$

Initially the exercise of drawing these pictures (circles and children) and figuring out different ways of sharing the rotis equally, engaged the children because often they had

no clue about how to do this systematically. They wrote the equations rather mechanically as the equations did not mean anything to them. With more experience children were able to recognize the difference in size and the need for naming the pieces. Even though we did not introduce unit fractions explicitly, children used unit fractions to name the pieces. If one roti is cut into 6 equal pieces and shared among 6 children, then they would say ‘each child gets $\frac{1}{6}$ (of a roti)’. If five rotis were shared equally among 6 children, the share each child could be either ‘ $\frac{1}{2} + \frac{1}{3}$ ’ or five $\frac{1}{6}$ pieces depending on how the sharing was done. Since they know that the share becomes smaller as the number of children increase, they could easily say $\frac{1}{6}$ is bigger than $\frac{1}{7}$ which is bigger than $\frac{1}{8}$ and so on. However, *our attempts at getting children to say that $\frac{1}{6}$ is the share of each child when 5 rotis are equally shared among 6 children were not always successful*. Most often the fraction symbol $\frac{m}{n}$ represented for them a sharing situation. To answer how much each got, they had to draw supporting pictures, divide and distribute and depending upon how they divided, say ‘ $\frac{1}{2}$ and $\frac{1}{3}$ ’ or ‘five $\frac{1}{6}$ ’. It is only very rarely they used the definition to answer how much each child got. We introduced the measure interpretation of fractions at this point because by then they had a ‘realistic’ though incomplete understanding of the fraction symbol $\frac{m}{n}$ and knew what the unit fractions represented. The measure interpretation reinforced their understanding of the fraction $\frac{m}{n}$ as m pieces of $\frac{1}{n}$ and enabled them answer the ‘how much’ question. At the NGO, Muskan, we used a linear scale and children marked out subunits and cut the whole into subunits and used these to measure their height, or length of various objects. At the school we used the ready-made circular fraction kit and children were asked to show different fractions using the kit, check equivalence or figure out for example how much a $\frac{1}{2}$ piece and $\frac{1}{3}$ piece together make.

Comparing Fractions

A graded approach beginning with comparing a fraction to 1, then $\frac{1}{2}$, then comparing fractions with the same denominator, comparing fractions with same numerator and finally comparing any two fractions revealed that at each level, children used their own reasoning to compare fractions.. While most of the students used the share interpretation, a few used the measure interpretation to compare fractions.

Some examples of students reasoning in comparison tasks:

1. Bittu (grade 5, from the NGO) compares $\frac{2}{5}$ and $\frac{1}{2}$ and says $\frac{2}{5}$ is smaller than $\frac{1}{2}$ because if we divide the 2 rotis into halves we will get only 4 shares and so to share equally among 5 children we must remove some small amount from each of these $\frac{1}{2}$ pieces. So each child would

get less than $\frac{1}{2}$ roti.

2. Vaibhav (grade 6, from the school) compares $\frac{5}{9}$ with $\frac{1}{2}$ and says it is more than $\frac{1}{2}$ because, if 5 rotis are divided into $\frac{1}{2}$ we will get 10 pieces and since there are only 9 children, one piece would remain which will again have to be shared among the 9 children so they will get more than $\frac{1}{2}$.
3. Shubham (grade 5, from the school) uses the measure interpretation and says $\frac{5}{9} + \frac{4}{9} = 1$ and $\frac{5}{9}$ is greater than $\frac{4}{9}$. Therefore $\frac{5}{9}$ is greater than $\frac{1}{2}$. 1. He also compares $\frac{11}{24}$ with $\frac{1}{2}$ as follows: $\frac{11}{24} + \frac{11}{24} = \frac{22}{24}$ which is less than 1. Therefore $\frac{11}{24}$ is less than $\frac{1}{2}$.

Though they would answer questions like ‘which is bigger $\frac{12}{23}$ or $\frac{13}{22}$ ’ by saying $\frac{13}{22}$ because we are sharing more rotis among fewer children, they often carried out an elaborate process of sharing either with the aid of pictures or in their mind. Also the method that they used for comparing two fractions might vary from day to day. For example, Akshay (grade 5, from the NGO) when asked to arrange the fraction $\frac{3}{7}$, $\frac{12}{7}$, $\frac{3}{7}$, $\frac{9}{7}$ in the increasing order writes them down as $\frac{3}{7}$, $\frac{3}{7}$, $\frac{9}{7}$ and $\frac{12}{7}$. When asked to explain how he did this, he gives an elaborate explanation, dividing rotis into halves counting how many remain after distribution and so on, while on another occasion he would have explained that the number of children remains the same and the number of rotis changes. So if the number of rotis increases the share of each child would automatically increase.

1. Samiksha (grade 6, from the school) compares $\frac{3}{4}$ and $\frac{4}{5}$ by drawing pictures, dividing each roti into same number of parts (say 4 in this situation) and distributing. After 4 rotis were divided into 4 pieces each and each child was given 3 pieces, she divides the remaining one piece into 5 equal parts and distributes and concludes erroneously that each child gets $\frac{3}{4} + \frac{1}{5}$. But when pointed out that she is sharing only $\frac{1}{4}$ of a roti into 5 equal parts, she reasons that if each quarter is divided into 5 equal parts there will be 20 pieces and so the extra piece is $\frac{1}{20}$ and writes $\frac{4}{5} = \frac{3}{4} + \frac{1}{20}$.

Sometimes they employed logical argument without the

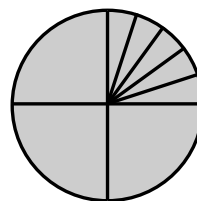


Fig. 2. Dividing the $\frac{1}{4}$ th piece into 5 equal parts

supporting pictures:

2. Bittu (grade 5, from the NGO) compares $\frac{1}{8}$ and $\frac{1}{9}$ as follows: from the 17 rotis if we give 2 rotis each to the 8 children, one roti will remain which if we divide equally among the 8 children they will get $\frac{1}{8}$ more; from the 19 rotis if we give 2 rotis each to the 9 children again one roti will remain which if we divide equally among the 9 children they will get $\frac{1}{9}$ more. Since $\frac{1}{8}$ is bigger than $\frac{1}{9}$, $\frac{1}{8}$ is bigger than $\frac{1}{9}$.
3. Goli (grade 5, from the NGO) concludes that $\frac{3}{8} = \frac{10}{16}$ by first sharing 4 rotis among 8 and giving each child $\frac{1}{2}$ roti and then distributing a $\frac{1}{8}$ piece to each from the remaining 1 roti. Similarly he first shares 8 rotis among 16 by giving each child $\frac{1}{2}$ roti and then divides each of the two remaining rotis into 8 equal parts and distributes a $\frac{1}{8}$ piece to each. In both the situation each child gets $\frac{1}{2} + \frac{1}{8}$ and so $\frac{3}{8} = \frac{10}{16}$.

It is interesting to note that on an earlier occasion when students were asked to compare $\frac{24}{36}$ and $\frac{21}{28}$, Goli reduced these fractions to $\frac{2}{3}$ and $\frac{3}{4}$ respectively by arguing that sharing 24 rotis among 36 children is the same as sharing 4 rotis among 6 which is the same as sharing 2 rotis among 3 and similarly sharing 21 rotis among 28 is the same as sharing 3 rotis among 4.

Later we reversed the situation and asked children to figure out how many rotis would be required for a given number of children if each child were to get a specified share. While these exercise were difficult because they could not draw pictures to support, they learnt to answer them, as long as the fractional part was $\frac{1}{2}$ or $\frac{1}{4}$ or at least a unit fraction. However when asked 'if each child were to get a share of $\frac{2}{3}$ rotis and if there are 12 children how many rotis would we require' Manisha (grade 5, from the school) took a leap leaving her classmates behind and said we would require 8 rotis because we have multiplied the number of children by a factor of four and so we should multiply the number of rotis also by a factor of 4.

Equivalence of Fractions

Introducing equivalence was easier with the share interpretation than with the measure interpretation. Children were introduced to equivalence in two different ways. One was to reduce a fraction like $\frac{24}{36}$ by discussing possible seating arrangements when 24 rotis are to be shared among 36 children. Children would distribute 24 into two equal parts and do the same with 36 and continue this process till the fraction is in the reduced form. They would get a set of equations:

$$\frac{24}{36} = \frac{12}{18} = \frac{4}{6} = \frac{2}{3}$$

The other method was to figure out in how many different situations one could get a certain share, for example $\frac{1}{2}$. Children could see readily that one could get $\frac{1}{2}$ in many different situations. The teacher wrote down the students' responses in the form of an equation like

$$\frac{1}{2} = \frac{2}{4} = \frac{10}{20} = \frac{7}{14} = \frac{30}{60} = \dots$$

Soon the children noticed that to get an equivalent fraction one need to multiply the numerator and denominator by the same number. That $\frac{2}{3} = \frac{4}{6} = \frac{10}{15}$ was something that they arrived at by multiplying the numerator and denominator by the same number and verifying by logical reasoning that they would be equal. In a typical classroom, this fact is something that the teacher would have told the children.

Addition of Fractions

We introduced addition of fractions such as $\frac{1}{2}$ and $\frac{1}{3}$ to children at the NGO by the following narrative:

Suppose there were a set of children and some rotis were shared equally among them. First each child got $\frac{1}{2}$ roti and $\frac{1}{3}$ then some more rotis came and were divided equally among the same set of children and this time each got $\frac{1}{3}$ roti. What could have been the sharing situation? Children were asked to list different situations in which one could get $\frac{1}{2}$ and they wrote down

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \dots$$

Next they were asked to list the situations in which each child could have got $\frac{1}{3}$ and they wrote down

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \dots$$

Then it was pointed out to them that if a child got $\frac{1}{2}$ and $\frac{1}{3}$ then one possible situation is that there were 6 children and first 3 rotis were shared equally among them and then two more rotis arrived which again were equally shared among the six children. Since 5 rotis were shared equally among 6 children,

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

At the time of instruction one of the students, Tasveer (grade 5) pointed out that another possible situation is $\frac{2}{6}$ and $\frac{1}{3}$ and so $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$. However, children did not really fol-

low the narrative, but *figured out for themselves* what they need to do in order to add two fractions: *for the two fractions one should write equivalent fractions till such time as one gets the same denominator and then one should add the numerators.*

Our repeated attempts to make them describe the process as we explained, failed but most of them learnt to add fractions. A couple of students took a leap *by figuring out the algorithm on their own* as Pappu (grade 5) and Tasveer (grade 5) did.

Pappu gave himself the following addition problem: $\frac{6}{8} + \frac{6}{9}$ and wrote down quickly

$$\frac{6}{8} + \frac{6}{9} = \frac{(6 \times 9)}{72} + \frac{(6 \times 8)}{72} = \frac{54 + 48}{72} = \frac{102}{72}$$

When asked to explain he said, ‘I recited the 8 tables and stopped at 72 because I thought the other denominator also might become 72 when multiplied, then wrote 9 in the numerator above (to keep track of the fact that $8 \times 9 = 72$ and so if 72 comes out to be the common denominator, the numerator also should be multiplied by 9), then recited 9 tables and found that 9×8 is also 72. So I wrote 8 in the numerator of the second fraction. Then I knew I was on the right track, so I multiplied 6 and 9 and similarly 6 and 8 and added the results’. Similarly Tasveer adds $\frac{3}{5}$ and $\frac{7}{7}$ by reciting the table for 5 and stopping at $5 \times 7 = 35$, saying may be this would be a common denominator and proceeds exactly like how Pappu did. However, immediately after this, Pappu added $\frac{1}{2}$ and $\frac{1}{4}$ by taking 8 as the common denominator and got $\frac{6}{8}$, which he said was equal to $\frac{3}{4}$. When asked ‘so how much is $\frac{1}{2} + \frac{1}{4}$, he said 3 rotis shared among 4 children. When asked to explain how much it is, he drew three rotis and 4 children and said $\frac{1}{2}$ and $\frac{1}{4}$. This whole exercise had a very humbling effect on us because we never expected that if we did not supply them the readymade algorithm, they would on their own figure it out and proceed to solve numerical problems leaving aside the conceptual issues that are too hard for them to comprehend just now.

Conclusion

We found that with the combination of share and measure interpretation, children could relate to the fraction symbol in a meaningful way and employ a variety of methods to compare fractions. Clearly for the children in our study, the symbol $\frac{m}{n}$ did not denote a pair of unrelated numbers written one above the other, but a *situation model* and a certain quantity. They could also generate a large number of fractions equivalent to a given fraction, by referring to sharing

situations and generalized from there, that to get equivalent fractions one should multiply the numerator and the denominator by the same number. From our attempt to give a conceptual understanding of addition of fractions, they figured out a method to add fractions. Because children had a situation model in mind when they saw a fraction, it was possible to encourage them to think on their own and step in and support or explain only when they needed help. This also meant there was much scope for children to follow each other’s thinking process, agree or disagree with each other’s reasoning and thereby enhance their conceptual grasp, though such an approach demanded more time than what is usually slotted for teaching fraction in the school syllabus. This being our first attempt at introducing fractions with the combination of share and measure interpretations, we did not do a systematic comparison of the two groups of children we studied. But going by their responses, we feel *there is no significant difference* between the two groups in their performance. It is also worth mentioning that with the share interpretation, activity based learning was possible at no extra cost or effort: children drew the pictures themselves and worked with them. We need to replicate this trial a few more times and through school teachers; we also need to build on what we have done so far and work towards an unified understanding of fractions incorporating operator and ratio sub constructs.

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