Using Research on English to Understand Mathematical Writing

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As a secondary teacher, I was in a unique position, for I taught both English and mathematics to high school students. Through my observations, I learned that my students employed similar writing practices when they learned mathematical skills. This study explored how high school students addressed the audience when they wrote mathematically.

Research Questions

I noticed various aspects of mathematics and writing that were similar. For this study, I chose to focus on one aspect: the role of the audience in the students' writing. As a result, my research questions were:

- Does the students' audience affect the kind of writing students produce in the mathematics class?
- If audience does have an affect on the students' mathematical writing, how is it affected?

Current Foundations of Mathematical Writing

In its essence, writing is the production of text. However, the process of producing the text is multifaceted. First is the acknowledgement of text. Vygotsky (1978) saw writing as a manifestation of spoken text. Taking it one step further, written text can be defined as symbols or signs that convey meaning (Rotman, 2000; Harris, 1995). These symbols can take on many forms, such as letters, numbers, mathematical and chemical signs, and musical notes. For each of these symbols, there is a specific meaning. Meanings, however, are not arbitrary; they are determined by the environment and social practices of its users. Once the meanings of the symbols have been established and acknowledged, adolescents need to be able to understand these combinations of symbols, in order for meaning to be conveyed.

According to Morgan (1998), mathematical writing is multidimensional, taking on more than one form. She explained that when students were able to use mathematical language properly, they were deemed "more literate in the topic" (p. 109). On the other hand, those students with either poorer writing skills or who used unconventional methods had little grasp of mathematical concepts. Morgan's concept of writing is similar to Gee's concepts of academic Discourse and discourse (1996, 2002). With both concepts, Discourse (with an upper-case D) is the recognition of an identity that is engaged in a particular activity, and discourse (with a lower case d) is the language used in the activity. In Morgan's case, the particular activity is writing and solving mathematical problems and the tools or objects that are used in this particular activity are mathematical symbols.

High school students are primarily taught mathematical writing through their mathematics teacher. The mathematics teacher is the authoritative voice in the classroom. Through the teacher's instruction, students will be exposed to how the mathematics teacher uses writing in the classroom. In turn, the student will try to emulate the teacher's mathematical writing usage. In my experience, these students were conscious of what was expected of them, because they equated conformity with a positive grade.

Methodology

This study was a mixed methods investigation of students'

mathematical writing and how it changed in relationship to audience. I used qualitative methods in collecting my data, and I used content analysis to identify qualities of writing that various clusters of writing samples appeared to share, examining writing samples that seemed to share common characteristics or qualities.

Research Participants

Thirteen students participated in this study who attended a suburban, parochial high school. They were Caucasian-American high school sophomores (grade 10), between the ages of 15 and 16 years old. 62% of the students were male, and 38% were female. These sophomores were the only participants in my study, because they had either Mr. M. (the mathematics teacher) or Mr. E. (the English teacher) as instructors. Five of these students only had Mr. M. as their teacher; four students only had Mr. E. as their teacher, and four students had Mr. M. and Mr. E. as their teachers. These thirteen students were evenly distributed between Mr. M's and Mr. E's classes.

Sophomore students were used because Mr. M. only taught sophomores. Mr. M. and Mr. E. were chosen because they were the most qualified. In the mathematics department, many of the teachers did not have proper content area certification. Thus, both were established teachers in their respective fields, having proper certification and pedagogical experience to teach their content areas. Within the English department, many were not comfortable giving a mathematical writing assignment as part of their daily routine classroom instruction. Mr. E. felt comfortable. As a result, only Mr. M. and Mr. E. agreed to be a part of this study.

Writing Prompts

Students were given writing prompts which I designed in both the mathematics and English classes. Two of the prompts were written in Mr. M's style, two were written in Mr. E's style, and two were written in the style of a friend. All of Mr. M's writing prompts followed the same format as his in-class writing assignments: a space for the student's name in the upper right hand corner, the word "worksheet" in its title, the mathematical problem to be solved in the center of the page, and the phrase "show work" in the worksheet's instruction. Writing prompts in Mr. E's class followed his format: a space for the student's name in the upper left hand corner, the title "In-Class Writing Assignment" centered at the top of the page, and the topic of the writing assignment (i.e., mathematical problem and its explanation) below the title.

Writing prompts to a friend took on two forms. The first form was in the format of an informal note, typical of what high school students give to one another. This prompt was handwritten, on lined notebook paper, with casual language. In the second form, it took on the format of an electronic mail message. The language was still informal, similar to what adolescents would send when seeking help on their homework.

Procedures

After obtaining permission from the students to examine their writing, I gave the teachers several directives. The first was to distribute the writing prompts, which I had written, as part of their routine, in-class writing assignment. I told them that they could use them in any manner, but I did not tell them how to present the writing prompts. Also, I asked them not to tell the students that the writing prompts came from me. In fact, I specifically stated, "Incorporate them [writing prompts] as naturally as possible."

All of the writing prompts were given two or three days apart, within the same week, over a four week period. Once they were completed, I asked the teachers to give me Xeroxed copies of all of the students' responses to the writing prompts before they graded the papers. Once I received the copies, I asked the teachers to grade the writing prompts as part of their routine assessment. I did not give the teachers a grading guide or rubric. Rather, I wanted the teachers to assess the writing samples as if it was part of the class' routine instruction, thus making the data collection process as natural as possible.

The first set of writing prompts was given in the first week of the study. The English class was given one writing prompt, and the mathematics teacher received two: One was written as if the teacher wrote the assignment, and the other was written as if a friend wrote a personal note to the student. Using the writing prompts as review exercises for an upcoming test, Mr. M. agreed to not distribute the prompts two days in a row. He agreed to give the prompts, skipping one day between them. Two weeks later, another set of writing prompts were administered. In this case, the mathematics teacher received one writing prompt, and the English teacher received two writing prompts: One was written in the style of the teacher, and the other was written in the style of an electronic mail message. Just as with the mathematics teacher, the English teacher agreed to give the prompts, as routine in-class writing assignment, skipping one day between them.

During the third week, I returned to the school and videotaped the teachers during their instructional lesson. In the

videotaping, I focused on the teachers' routine classroom instruction. Primarily, I wanted to record how they used writing in their routine instruction. At the same time, I wanted to understand the students' learning environment for each class. As a result, when there were no students in the room, I videotaped the walls of each classroom. I wanted to know all of the kinds of writing that the students encountered during their mathematics and English lessons.

Data and Analysis

Since this study explored the nature of students' mathematical writing and how it changed according to audience, I devised a coding scheme that allowed me to classify the students' writing into different categories. First, after examining the writing samples, I noticed two main types of writing and clustered them into two groups: mathematical writing with equations only and mathematical writing with equations and scripted longhand. Then, I examined each cluster and noticed that the cluster of mathematical writing with equations only all looked similar. All of these samples had a series of equations with "=" connecting the two parts of the equations. On the other hand, the cluster of mathematical writing with equations and scripted words could be further categorized into two different sub-types: Code Switching and Multiliterate Hybridity. Once I identified the types, I compared the students' writing samples to the qualities of writing the students encountered in their instructional environment. Focusing primarily on the scripted words that accompanied the equations, it was clear to me that the students' writing samples displayed distinctive qualities.

Of the 58 writing samples, 43 writing samples (i.e., 74%) had numbers and mathematical symbols only, in the form of equations. Fifteen writing samples (i.e., 26%) had equations and scripted longhand; these samples had equations on one part of the page and verbal sentences on the other part of the page. There were one to three sentences per writing sample. None of these writing samples contained complete paragraphs. However, all 15 writing samples came from students who completed the writing prompt in English class.

Also, four students had the same mathematics and English teachers. Of these four students, three of them wrote differently for their English teacher, as compared to their mathematics teacher. The writing samples collected from their English teacher contained more scripted longhand words in addition to mathematical equations than the writing samples collected from the mathematics teacher, which only contained mathematical equations. In fact, none of these three students wrote scripted longhand words in the writing samples collected from their mathematics teacher. They only wrote mathematical equations and symbols in response to prompts assigned by their mathematics teacher.

Writing samples collected in the mathematics classes reflected the writing presented by the mathematics teacher. The mathematics teacher used numbers, logograms, and pictograms in his routine teaching. Also, instead of using traditional scripted longhand sentences, he used equations. The writing samples collected from the students in the mathematics class reflected this style of writing. All of Mr. M.'s students wrote numbers, logograms, and pictograms in the form of equations in their mathematical writing. They did not use scripted longhand in any of their mathematical writing. All students followed the pattern.

Writing samples collected in the English class reflected the writing that was presented by the English teacher. Mr. E. used scripted longhand in his routine instruction. When he gave a worksheet or writing prompts, he used words and complete sentences. The student writing reflected this presentation. There were nine writing samples that consisted of scripted longhand and eight writing samples that consisted of only numbers, logograms, and pictograms, in the form of equations. The writing samples that contained scripted longhand were reflective of the writing, which was presented in class. These writing samples contained complete sentences that either defined or explained the mathematics that the students were asked to solve. Three students followed the pattern, and one student did not.

Writing prompts that were written as a friend were given in both the mathematics classes and the English classes. Then, writing samples did not reflect the style of writing in the prompt. Instead of using slang and other idiomatic expressions, these writing samples used more formal language, and they reflected the writing that was presented in the mathematics and English classes. There was no difference between these writing samples and the writing samples from the prompts that were written in the style of the teachers.

Four students in this study received instruction from both Mr. M. and Mr. E., and they received writing prompts from both teachers as part of their routine instruction. There were a total of 17 writing samples from this sample of students. In this population, three of the four students changed their mathematical writing according to their audience. These three students produced 12 writing samples. Within these 12 writing samples, six were collected in the mathematics class, and six were collected in the English class. The six writing samples that were collected in the mathematics class were reflective of the writing that was presented by Mr. M. In the English class, six writing samples were collected,

and they contained scripted longhand in addition to equations. The scripted longhand contained complete sentences, reflecting the style similar to what the students were taught in their English class.

This group, who had both Mr. E. and Mr. M. as teachers, showed that the students did change their mathematical writing as their audience changed. They wrote only equations to Mr. M., but when they wrote to Mr. E., there were scripted longhand sentences in addition to the equations. Scripted longhand sentences were the expectation in Mr. E.'s class, and they gave it to him.

Students were given three different kinds of writing prompts. One was written in the style of the mathematics teacher, one was in the style of the English teacher, and one was written in the style of a friend. After I collected the writing samples, I noticed that students changed their writing style for the mathematics teacher prompt and for the English teacher prompt. However, students did not adjust their writing to the friend prompt. Instead, they wrote as if their mathematics teacher or their English teacher was reading the prompt. Mr. M. told me that he expected the writing to change for the friend prompt and was surprised that his students wrote in the same manner. Through informal discussions, I learned that Mr. M. asked his students what they thought about the writing prompt that was written in the style of a friend. His anecdotal evidence revealed that his students perceived the prompt written as a friend as another in-class worksheet, and they knew they were getting graded for it. Mr. M. also told me that his students knew that he, the teacher, would be the only person grading their work and that my reading the writing prompts were inconsequential to them.

Discussion

As stated earlier, the writing samples were divided into two categories: writing that consisted of only equations and writing that consisted of equations and scripted words. Between these two major categories, three types of mathematical writing emerged. I call them: Separated Writing, Code Switching, and Multiliterate Hybridity.

Separated Writing

Separated Writing consists of only numbers, logograms, and other mathematical symbols, in the form of an equation. It uses one form of mathematical language to explain the problem solving processes. In this case, the students only used equations, which could be seen as monolingual, using only one kind of mathematical language: numbers, logograms, and pictograms. 43 writing samples had this kind of writing, and all of these writing samples came from the mathematics class.

There are several possible reasons why Separated Writing was primarily seen in the mathematics class. The first possible reason is that the students were writing in a language that was expected of them. Mr. M. did not use much scripted longhand in his teaching, and the scripted longhand that he did use was specifically mathematical in nature. For example, when he wrote on the chalkboard, he wrote: "if," "then," "also," and "from." Even when he wrote the homework assignment on the chalkboard, at the end of class, he wrote a series of numbers that signified the problems to be solved in their textbook. He did not write any scripted longhand explanatory information about the methodology or details about the individual homework problems. As a result, the only mathematical writing the students understood from Mr. M. was the use of numbers, logograms and pictograms.

A second possible reason could be the mathematical fluency of the students themselves. Goldsby and Cozza (2002) found that as students became more knowledgeable in the mathematical processes, they were able to explain those processes better. These researchers also found that a student was able to solve a problem, but she/he may not have been able to explain the process in which she/he solved it. The same may be true in Mr. M.'s class.

Code Switching

I coined the term Code Switching to this second type of writing, because students used two different kinds of mathematical language, scripted words and equations on the same page. Intermediate writing gives the reader some idea that the student understands the mathematical concepts involved in solving the problem. Since the student is able to communicate either a definition or explanation of the mathematical concepts that are expressed in the problem, it is clear that this student is more knowledgeable in the mathematical processes (Goldsby & Cozza, 2002). 9 writing samples had this type of writing, and all of these samples came from the English class.

Code Switching uses numbers and mathematical symbols as nouns, within the written text. The student uses subject specific lexicon to explain the mathematical process. Mathematical lexicon consists of numbers, mathematical symbols, and mathematical vocabulary and definitions. Numbers and mathematical symbols are used as nouns, as a form of shorthand. For example, the mathematical verbs are written into words ("bisects," "equals," "divides"), and the use of linking verbs (is) and action verbs (make, solve)

have mathematical connotations. The numbers and mathematical symbols are used as subjects, objects, or predicate nominatives, within the scripted longhand sentences. They are not part of a mathematical equation. For example, one student wrote "then, m∠PQS is also 27." Here, "m∠PQS" is the subject, and "27" is the predicate nominative. The meaning of this clause is the same as the equation. The only difference is that the equation has "=" as its linking verb. In the scripted longhand writing, the mathematical notation is used as a form of shorthand. Instead of writing "the measure of the angle PQS," this student wrote "m∠PQS".

Code Switching is commonly seen in journals and learning log entries. When students try to convey their metacognitive and affective processes, they may explicate their emotions and thoughts in scripted longhand. The mathematics, in question, is separate from the actual scripted longhand text (Borasi & Rose, 1989). This kind of mathematical writing uses numbers and mathematical symbols as nouns, subjects and objects in the text. Equations are not necessarily used to designate complete sentences. The reader has the impression that the student has some grasp of the language, but that same student may not be fluent enough to use both languages interchangeably. Rather, this student may only be able to use isolated words and ideas of the new language and incorporate what she/he knows within the current language. In Code Switching, the new language is mathematical language and the current language is English.

Multiliterate Hybridity

Multiliterate Hybridity is similar to Code Switching, because both kinds of writing consist of scripted longhand and enumeration. However, with Multiliterate Hybridity, the scripted longhand and enumeration are entwined, in a sense, to explain the mathematical processes. Mathematical equations are incorporated and used as individual sentences as part of the explanatory process. For instance, in one writing sample, a student numerically solved the problem. Then, off to the side of the paper, he wrote: You need to add the 3y and 96 so it equalls [sic] 180^{O}. Then minus 96 to the 180 and the 96 to make it a zero [sic]. So then you need to divide 84 by 3 then $y=28^{O}$. While this student's writing is not grammatically correct, he did use mathematical symbols as predicates (y=28), and incorporated mathematical text and language in his explanation. 6 writing samples had this type of writing, and all of the samples came from the English class.

The ability to incorporate scripted longhand and mathematical language is the most multiliterate form of mathematical writing, because it shows complete incorporation of two languages (mathematical language and English) in a single coherent form. With Multiliterate Hybridity, it is clear that the student has a clear grasp of the mathematical concept.

This Multiliterate Hybridity is seen in professional mathematics journal articles. For example, mathematicians manipulate language to clarify and explain their mathematical process to the mathematical Discourse community (Burton & Morgan, 2000). In a crude way, the students act similarly. In this case, 3 of the 13 students manipulated language within the mathematical Discourse, incorporating both scripted longhand and mathematical equations to clearly explain a mathematical process. Multiliterate Hybridity also shows how students are engaged in the meaning-making process of mathematics. For this process to be effective, students took what was interpreted (in this case the mathematics) and incorporated that information into their own lexicon. Bahktin (1935/1981) stated that this interpreting process takes place within the specific socio-cultural context. This juxtaposition of language shows duality, the ability to flit back and forth between languages, with relative ease and coherency. Also, these students used mathematical concepts in mathematical situations. The student is able to incorporate both kinds of mathematical language, equations and scripted longhand, into a coherent explanation. While it may not be clear that the student is truly fluent in mathematical language, what is clear is that she/he has some command of it and is able to use them accordingly. Upon informal discussions with their mathematics teacher, Mr. M. mentioned the names of his best students, and I realized that these students coincided with those who used Multiliterate Hybridity.

Conclusion

This study showed that mathematical writing existed in scripted longhand and mathematical symbols, logograms, and numbers. Neither form of mathematical writing was seen as better nor more important. Rather, both forms of mathematical writing were necessary to convey complete meaning in the student's mathematical writing.

In mathematics class, the students wrote mathematically through the use of equations. This kind of mathematical writing was reflective of what the students' mathematics teacher used during his routine classroom instruction, since he used few scripted words and mostly used numbers, logograms, and pictograms in his instruction. When the students wrote mathematically in the English class, a different kind of mathematical writing emerged. Here, in addition to equations, students used scripted longhand, words and scripted sentences, to explain their mathematical ideas. Also, these students used numbers, logograms, and pictograms as nouns and verbs, embedding these mathematical sym-

bols into the scripted words. Students in the English class were exposed to scripted longhand during their routine instruction, and all of their instructional materials contained this kind of writing. Thus, the students' mathematical writing reflected the English teacher's writing.

This study showed that students do attend to audience by adapting to the kind of writing they were exposed to in their routine classroom instruction. If the mathematics audience implies the use of numbers, mathematical symbols, and logograms, then the students will follow suit. If scripted longhand is taught only in English classes, then writing in the different disciplines will not get developed. Three modes of mathematical writing emerged in this study: mathematical writing that consists of mathematical symbols and numbers in the form of equations only, mathematical writing that consists of words only, and mathematical writing that is a hybridization of both words and mathematical symbols and numbers. In the learning of mathematics, students need to learn how to manipulate the various modes of communication. These modes include mathematical signs, mathematical symbols, and subject specific vocabulary. If students use only one mode of mathematical communication, either scripted words or equations only, they are able to convey mathematical meaning, but that meaning may be limited. By using the full range of communication tools, students show that they understand and can manipulate the full range of mathematical language. When students intertwine equations and scripted longhand into complete sentences within one document, a more comprehensive meaning emerges. Students not only understand the basis of the mathematics, but these same students understand how mathematical language is manipulated to convey their mathematical meanings (Burton & Morgan, 2000; Goldsby & Cozza, 2002).

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