Virtual Manipulatives: Potential Instructional Hazards and Possible Design-based Solutions

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Virtual manipulatives are employed by both preservice and inservice teachers to enhance the instructional effectiveness of physical manipulatives and related tools by addressing limitations of access, cost, and adaptability. While research into the use of emerging technologies continues, there are several variables to consider when using, or measuring the effects of virtual manipulative use. Research design, sampling characteristics, and the type of manipulative used may influence achievement. For example, some studies that have shown evidence of increased achievement were administered when classroom teachers believed they fit in with the natural flow of the curriculum. Other studies with no noticeable increase in student achievement were administered at times that interrupted the normal curriculum. Other variables that may influence the effectiveness of using virtual manipulatives include: previous experience with computers, grade level, mathematical topic, treatment length, student attitudes toward mathematics, and computer-to-student ratio.

Introduction

One of the pedagogical techniques in mathematics education is to provide students opportunities to actively manipulate certain aspects of the phenomenological world (Heddens & Speer, 2008; Izydorczak, 2003; Moreno, 2005; NCTM, 2000; Olson, 1988). This technique relies on careful construction of those phenomena that exemplify the mathematical concept being conveyed. In essence, these phenomena serve as concrete analogies of mathematical concepts and, in the language of mathematics education, are said to *model*[1] those concepts. Pedagogical tools specifically designed for this type of active manipulations are called "manipulatives." With the advent of digital technology, this basic idea of manipulatives has been extended to the computer-based manipulatives or "virtual manipulatives" (Schackow, 2007; Tversky & Morrison, 2002).

This paper identifies and discusses 1) some potentially detrimental effects that the use of virtual manipulatives may have on mathematical learning, and 2) possible ways to address these effects. Note, however, that the intent of this paper is not to dismiss pedagogical advantages that virtual manipulatives may afford; its intent is merely to discuss some potential pitfalls in their design and use.

The issues identified in this paper are based on observations of students interacting with a number of implementations of virtual manipulatives found on the Internet, namely, at the National Library of Virtual Manipulatives for Interactive Mathematics website. The findings in this paper are based on test cases or anecdotal evidence and, therefore, lack the necessary basis for definitive conclusions. This paper is presented as a preliminary study upon which further, more rigorous, investigation may be formulated.

Methodology

This paper is based on two different types of observations. In each, the observer made handwritten notes of events that were deemed noteworthy. The observer was not pre-

[1] It is curious that the notion of *modeling* in mathematics education is a mirror image of that found in science. In physics, for example, mathematical concepts are used to *model* physical phenomenon. Here, the object of study is the phenomenological world and mathematics is a language used to describe that world. In mathematics education, on the other hand, the object of study is mathematics itself and the phenomenological world is used to *model* mathematical concepts.

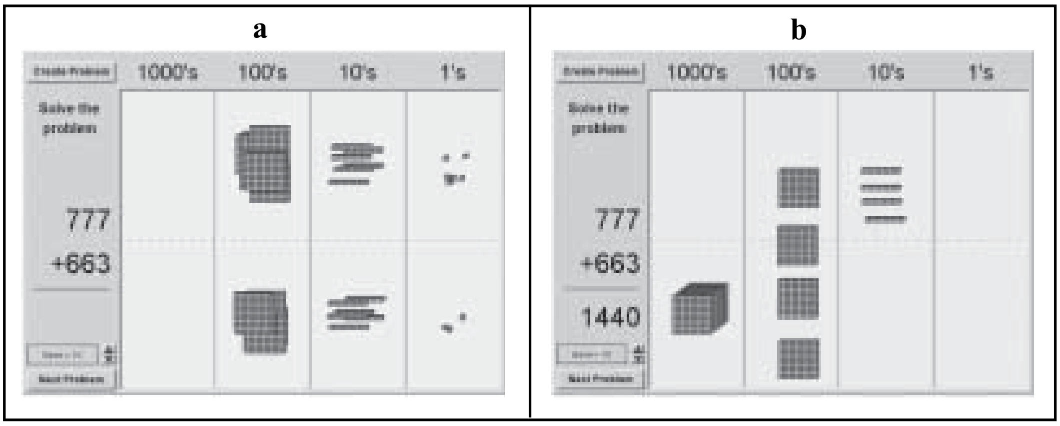

Fig. 1. Screen shots of virtual Base-10 Blocks manipulative

pared with any pre-planned set of questions or focus of observations. These observations are based on a series of sessions with 4th grade students for a duration of approximately one hour. During these sessions, the observer and the student explored the problem of integer addition using both the physical Base-10 Blocks manipulative and a virtual implementation of Base-10 Blocks. The sessions consisted of a series of problems on integral addition, where each problem was attempted, first with the virtual manipulative and, then with the physical manipulative (Mousavi, Low, & Sweller, 1995; Schnotz, 2005).

Figure 1a shows the screen shot of a typical problem with the virtual Base-10 Blocks manipulative. Here, the object of the manipulative is to 1) aggregate 10 pieces in one unit into a single piece in the next higher unit, and 2) place each piece in the columns to which they belong. The screenshot of the solution is presented in Figure 1b.

These observations are based on three "computer-lab" sessions of three different middle school mathematics classes. The three classes consisted of magnet-only, magnet-zone combined and zone-only student populations. Most of the students worked individually, each with a dedicated computer. However, due to a limited number of computers, a small minority of students worked in groups of two. Also, on occasion, especially when students appeared to be "stuck" in a particular scenario, the observer interjected with questions and suggestions to the students.

Each lab session began with the students exploring the Circle-0 virtual manipulative screen shot is shown in Figure 2a. The object of Circle-0 is to place all the numbers (using the drag-and-drop technique with the mouse) within the circles so as to make each circle to "add up to 0" (see Figure 2b).

Once they had tried Circle-0, the students were free, as time permitted, to try additional virtual manipulatives available at National Library of Virtual Manipulatives for Interactive Mathematics site. At the conclusion of the lab, students were required submit a written description of their experiences.

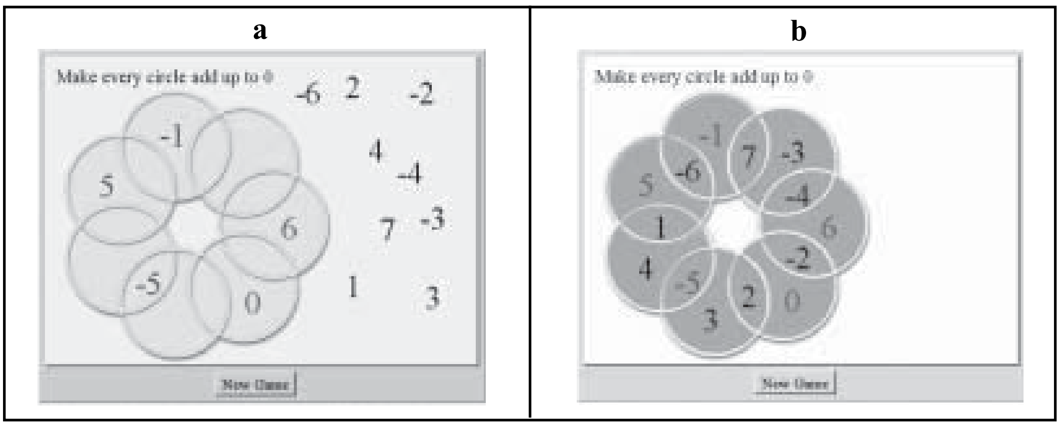

Fig. 2. Screen shots of Circle-0 virtual manipulative

Potential Issues

This section identifies 4 different ways virtual manipulatives may be counter-productive to learning. Each is discussed in terms of

- Characterization of the potential issue
- The observations that support this characterization
- Ramifications of this issue for effective and efficient learning
- Potential design solutions to address these issues
- Procedures devoid of concepts.

Under certain circumstances, manipulations may not be accompanied by their intended conceptual counterpart. Students may acquire procedural expertise needed to successfully complete the manipulatives without internalizing the mathematical concepts that the manipulatives were designed to model (Atkinson, 2002; Izydorczak, 2003).

An observation that prompted this issue was with a student's interaction with Base-10 Blocks. With virtual Base-10 Blocks, the procedure for combining 10 pieces in one unit required surrounding 10 *or more* pieces with a bounding rectangle with the mouse. Once those pieces were successfully bound, the computer automatically transformed 10 of those pieces into a single piece of the next higher unit. It became clear that the student became focused on the operation of surrounding all of the pieces in each unit with the bounding rectangle. From the student's perspective, this was a reasonable strategy for the mastery of that skill – irrespective of any concepts that may be associated with it, it was what was necessary to complete the exercise.

The fact that the student, when given the exact same problem that he had immediately before successfully solved using the virtual Base-10 Block, had difficulty replicating his solution with the physical Base-10 Blocks lent additional credence that, with this particular student, there was a chasm between his understanding of the procedural requirements of the manipulative and their corresponding meaning in the number system. With a clear understanding of the relationship between the two, one would expect that changing the medium of the manipulatives – from virtual to physical – would have had significantly less impact than was observed (Drickey, 2001).

Clearly, when students "play the game" devoid of the concepts the manipulative is designed to demonstrate, the effectiveness of the exercise in meeting the intended pedagogical goals is likely to be limited. However, this potential for procedural expertise devoid of conceptual understanding seems to be an inherent vulnerability of manipulatives in general. Manipulatives are, in essence, phenomenological analogies for concepts and, as such, always carry the possibility of being misconstrued. Of course, the task of educators is to limit the likelihood of those misconceptions and to resolve them when they do occur. However, in principle, the chasm between procedure and concepts cannot be eliminated (Large, et al., 1996).

What, then, are some potential strategies to 1) limit the likelihood of conceptually empty procedural manipulations, and 2) resolve them when they occur? With respect to the example cited above with virtual Base-10 Block, the virtual manipulative may be designed so that: the student must explicitly collect exactly 10 pieces of one unit rather than simply surround 10 or more pieces within a bounding rectangle; the student must explicitly request (say, using a button) to convert the 10 pieces to a single piece of the higher unit; and/or the student must replicate the manipulation with symbolic operations. In general, by requiring of the student greater responsibility of the manipulations (as opposed to automated manipulation by the system) and corresponding symbolic operations, the likelihood that the student will make the conceptual connection with procedural operations may be increased (Pass, Renkl, & Sweller, 2003).

Local Minimum

Some students seem to get "stuck" in a local minimum of the search space. As a result, those unwilling or unable to backtrack (i.e., give up some of the gains seemingly achieved) were unable to reach the global minimum (i.e. complete the problem) (Schnotz & Bannert, 2003). The level of challenge represented by these local minimums may be counter-productive to some students. This issue was observed particularly with the Circle-0 virtual manipulative. The screen shot in Figure 3a is an example of a local minimum.

In this example, 5 out of 7 circles (in yellow) have met the criteria, i.e. they "add up to 0". There are two numbers left (-2 and -4) which must be placed in the two remaining spots (between 7 and 6) so that the remaining two circles (in grey) also add up to 0. It is easy to see that neither of the two combinations of placing the remaining numbers produces the solution. Therefore, in order to reach the solution where all the circles add up to 0, the partial solution generated so far must be sacrificed. In other words, we must take out at least some of the numbers in the yellow circles, i.e. the circles that already add up to 0.

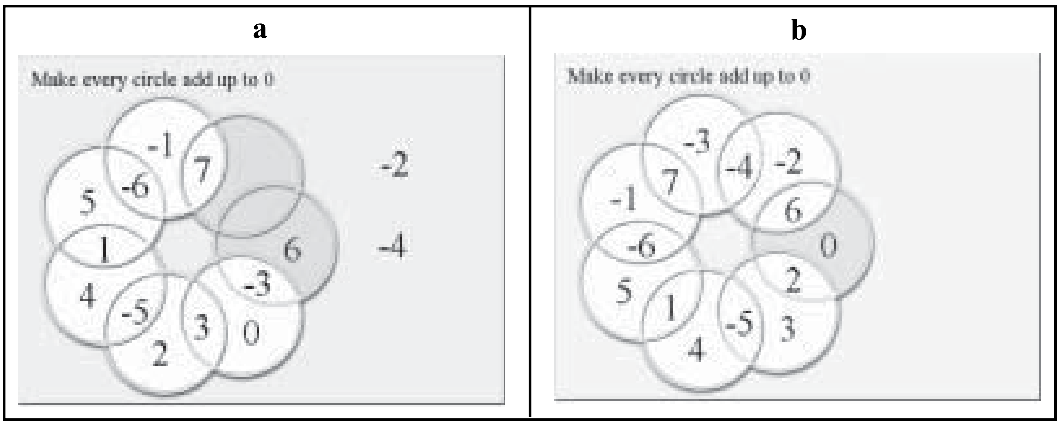

Fig. 3. Local minima in two Circle-0 solutions

The screen shot in Figure 3b provides an even more dramatic example of a local minimum. In this example, all of the numbers have been positioned; however, only 6 out of 7 circles add up to 0. The situation can be, it seems, rather perplexing to the student.

The question remains as to whether such perplexity is conducive or counter productive to learning. It seems that, in general, the answer depends on the student. In terms of Vygotsky's learning theory, the answer depends on the student's "zone of proximal development" (Schnotz & Rasch, 2005). In other words, for a sufficiently advanced student, the challenge posed by this situation may be appropriate while, for others, it may represent an inappropriate level of challenge. The detrimental effect of too much challenge seems particularly relevant in the area of self efficacy. Specifically within the context of Circle-0, however, the level of challenge posed by such false solutions (where the solution seems so close and yet so far) seems almost always inappropriate for students that the manipulative is target, i.e. students learning to add single digit integers.

One obvious way to control the potential level of local minimum that a problem contains is through additional constraints. In the case of Circle 0, additional starting numbers may be added inside the circles (see Figure 2a) so as to remove the possibility of deep local minimums. From a technical point of view, given the size of the search space and the computational speed of even modest computers today, it should be feasible to check all possible points on the search space so as to ensure maximum level of difficulty in terms of local minimums.

Disengagement

Some students were observed to be disengaged with the problem at hand. They repeatedly, for prolonged periods, hit either the "Hint" or the "Reset" button without demonstrating any attempts at actually solving the problem presented to them. To the observer, it was as if, once they had developed the pattern, they were stuck in a mental mode of simply pressing those buttons. Needless to say, such a lack of engagement with the problem is counter-productive to learning. Even with legitimate attempt at engaging the problem, over reliance on these scaffolding devices are counter-productive to the learning.

The problem of disengagement seems not a problem with virtual manipulatives, per se, but a manifestation of a more general issue in learning (Mayer & Chandler, 2001). A student's inability or unwillingness to effectively engage a problem likely points to more fundamental issues in learning and may require higher levels of intervention. Therefore, it seems unlikely that this type of disengagement would be eliminated by simple design changes in the manipulatives. At least in principle, however, it may be possible to enable the virtual manipulative system to automatically 1) detect certain patterns of use (or rather misuse) of the program and 2) to provide some type of interjection (or notification to a human instructor).

From a technological perspective, the level of computational sophistication needed for such functionality is qualitatively different than what is found in implementations of virtual manipulatives at the National Library of Virtual Manipulatives for Interactive Mathematics website (or anywhere else based on this author's observation). This type of functionality – automated diagnosis of student performance and feedback based on that diagnosis – has yet to be effectively demonstrated (or, perhaps, even discussed) in educational technology. It is the author's view that the development of this class of functionality will likely become a focus of research.

Rule Confusion

Some students seemed not to understand what the virtual manipulative was asking of them even after they spent some time reading the accompanying instructions. Some students were heard asking themselves, “Now what do I do?” or “How do I play this game?” On several occasions, the observer interjected by demonstrating how to “play the game”. In each instance, the students were able to engage the problem at hand (Mayer & Moreno, 1998). For example, one group of students seemed in a state of bewilderment with the Circle-0 manipulative. However, once they were shown how to fill in one or two circles, they immediately and quickly progressed with the remaining circles. Apparently, the light bulb had turned on. The amount of cognitive and emotional resources expanded during these periods of confusion by some students is likely counter-productive. Certainly, from the learner’s point of view, they are both unpleasant as well as unproductive.

One way to increase the understandability of the instruction may be to simplify the language of the instruction. The online instruction for the Circle-0 manipulative, for example, is shown in Figure 4.

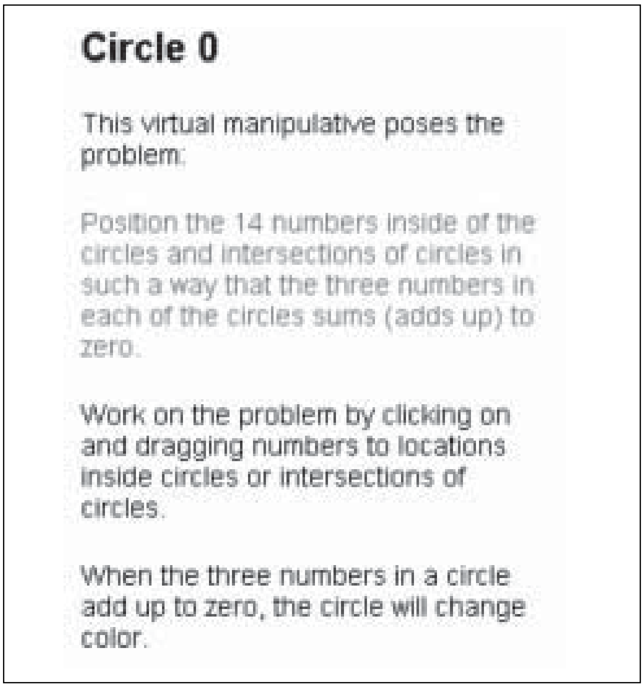

Fig. 4. Online instruction for Circle-0 virtual manipulative

Given their linguistic characteristics, it is unclear (at least to this author) for what kind of readers these instructions are intended. Are they written for elementary and middle school students that play the manipulative, or for a more sophisticated adult population like classroom teachers? Using a language more suitable to younger students may increase the likelihood of being understood.

An even more intuitive and effective way (albeit, more difficulty and costly as well) to convey the instructions for using the manipulatives may be through animated demonstration of it use (as opposed to written instruction alone). Animation may be effective in communicating not only basic instruction but also different strategies for tackling the problem (Atkinson, 2002).

Conclusion

Perhaps, it should be axiomatic that every technology has its limitations and, therefore, can be misused. This paper discussed some of the potential inherent in and the design limitations of virtual manipulatives and how these limitation may be addressed. One of the conclusions based on the observation cited in this paper may be that appropriate supervision is needed to maximize/minimize the potential benefit/detriment of virtual manipulatives. It is the view of this author that one of the major areas of research in educational technology is the development of assistive mechanisms to effectively and efficiently support this type of supervision.

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