“It Makes Sense to Ask That Question”: A Closer Look at Student Questions

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Several studies have investigated classroom mathematical discourse and how teachers and students participate in it to create meaningful interactions. In this paper I consider the questions that students ask in a high school math classroom in relation to the teacher's expectations of the kinds of student questions he wants to encourage in the discourse. Using the framework of reification, I show that there is a relationship between students developing a structural outlook to asking questions that the teacher expects from them.

Mathematical classroom discourse has been the topic of several research studies (Hufferd-Ackles et al., 2004; Lampert, 1990; Zack & Graves, 2001). These studies have elaborated the participation structures, the forms of mathematical talk and teacher roles (listener, facilitator, co-constructor, etc.) to promote meaningful mathematical discourse in the classroom. In such cases the nature of teacher-student interactions and the role of the teacher changes (Lampert, 1990; Rittenhouse, 1998; Zack & Graves, 2001). The teacher's role shifts from being didactic– the only one doing all the talk – to dialogic (Rittenhouse, 1998). This requires teachers to keenly attend to student talk and their questions as they make sense of the mathematics (Lampert, 1990; Schifter, 1996; Zack & Graves, 2001). Though challenging in practice, this can lead to interactions that support a deeper conceptual exploration and understanding of the mathematics. While several authors have investigated challenges and benefits of teachers attending to student thinking in classrooms (Carpenter et al., 1999; Lampert, 1990; Schifter, 1996), we know little about how teachers interpret student questions in relation to the mathematics being taught or learnt.

If we assume that student questions emerge from interpretations using their current understanding and, if teachers are in an endeavour to create rich mathematical discourse (that in turn supports students' mathematical understanding), we need to take a closer look at how questions emerging in the classroom are interpreted and used by teachers. How do teachers attend to questions that students ask, in relation to the type of questions they expect from students, within the context of the mathematical concepts being taught and learnt? In this paper I investigate an instance of one teacher's interpretations of student questions in relation to his expectations of the kinds of questions he wants his students to ask.

Situating the Study

This study is situated in an all-girl public charter school located in a large urban city in the United States. The school focuses on preparing women for college and career by encouraging engagement in math, science and technology and uses a well-known reform math curriculum namely, *Interactive Mathematics Project* (IMP) for its high school grades. The school is non-selective with 350 students in grades 7-12 and is 78% African American, 15% Latina, 6% Caucasian, 1% Mixed Race and 1% Asian. For 10 weeks, I was a participant-observer in one of the junior grade mathematics classroom with 20 girls: 13 African-American, 4 Latinas, 2 Caucasians & 1 Asian, and Mr. Benton Taye (a pseudonym), an African-American teacher. In Mr. Taye' classroom students regularly raise questions to him and their peers, present solutions, and hold him and their peers accountable for what he or they say.

During these 10 weeks, they were working on *Meadows or Malls?,* a Year 3 IMP module unit that extends the concepts of linear programming problems with two variables introduced in the Year 2 unit *Cookies*. Students develop a strategy for solving linear programming problems in more than two variables and solve systems of linear equations using the elimination method and matrix algebra. The students have to ultimately find a land allocation that satisfies the constraints specified in the unit problem – mathematically it is a linear programming problem in 6 variables and 12 constraints.

Before starting the classroom observations, I had a discussion with Mr. Taye to identify aspects of classroom discourse and student learning that he was interested in investigating further – one of which centred on student questioning. I took in-class field notes and audio-taped the classroom discussions. I also had one hour discussions with Mr. Taye once every 3-4 weeks on the data collected, which were audio-taped. Prior to these, I reviewed my field notes to identify a few questioning episodes to discuss with him. For some questions he had clearly expressed appreciation or surprise in class. During our discussions, I probed for his views on his questions, his response to them and the interactions that followed to get a better sense of how he interpreted and used them in his instruction and attended to student thinking expressed in these questions. Due to time constraints, I was unable to analyse the questions before discussing with him. In the following sections I highlight portions of my discussion with Mr. Taye on student questions and a transcript of classroom interaction, followed by an analysis of the data presented.

What Questions Did the Teacher Want?

During my first discussion with Mr. Taye, I (the researcher) asked him what kind of questions he expected from his students and how these questions would help him.

Mr. Taye: [in response to my question] Questions that provide evidence of them thinking a little bit deeper about the mathematics, making connections, questions that ask why things happens a certain way.

Researcher: How are the questions helping you?

Mr. Taye: It is letting me know that they are thinking about the stuff at a little bit deeper level than at the surface.

Researcher: So you can help them figure out what they are trying to make sense of.

Mr. Taye: I mean all questions will help me help them figure out whatever it is that they are asking that's the point of the question, right. I would rather have them ask me questions like why does something behave that way or uhm, what if this happens, or you know, questions that I could pose back to the other students that challenge their thinking, rather than questions that are like, uhm, something that they are asking because they didn't hear me say it the first time…. Other questions that ask them to stretch their thinking, extend their knowledge, or explain why something is the way it is.

Mr. Taye expected his students to ask questions that showed "evidence of them thinking a little bit deeper about the mathematics, making connections" and not "because they didn't hear" him the first time. During the course of this unit, I identified several questions and discussed with him in an attempt to elaborate what "thinking deeper about the mathematics," "making connections," looks like in student questions. I present an episode of classroom interaction and focus on two student questions from this episode for this purpose.

Questioning Episode

At the beginning of the unit, the students identified a set of six variables required for the unit problem – AR, MR, GR, AD, GD, & MD. Each variable represented an amount of land allocated for a specific purpose (recreation or development) from three different pools of available land (Army, Mining, Farm/Goodfellow). Each student identified constraint equations from the information given in the problem. In groups they compiled a list of equations. For e.g., they had a constraint equation $AD+MD+GD \geq 300$ to reflect the statement "at least 300 acres would go for development" and so on. After a few minutes of group work Mr. Taye asked each group to report out a constraint equation and how they got it. The transcript begins when he gets to Keisha's group.

1 *Keisha:* We did AR+GR+MR+GD+MD = 550 [Mr. Taye writing on the board], because they all equal, coz all together recreation and development is 550.

2 *Mr. Taye:* Okay, thanks, I am going to address that in a minute. Anything from this table?

[He goes around the remaining groups and gets one constraint from each group.]

3 *Mr. Taye:* This is definitely true [pointing to the constraint Keisha gave]. But for our purposes we need to break this one big equation into smaller equations. [pause] We are not going to use this one. But can make 3 smaller equations from this big equation. Perla, can u give me one?

4 *Perla:* GR + GD = 300.

5 *Mr. Taye:* GR [crosses out the big equation and starts writing this one one board], Can you explain that to us?

6 *Perla:* GR is for Goodfellow recreation and GD is for Goodfellow development.

7 *Mr. Taye:* So what are you saying?

8 *Perla:* That recreation and development got 300 acres from Goodfellow.

9 *Mr. Taye:* So essentially, all the farm land equals 300 acres.

…. [Two students give the other two equations AR + GR = 100, MR + MD =150].

10 *Mr. Taye:* There's actually 6 more constraints.

11 *Ss:* Really!

12 *Mr. Taye:* Not in your reading, it's just intuitive, just implied, meaning,

13 *Keisha:* [interrupting] I got a question.

14 *Mr. Taye:* [after a pause] Okay, Keisha, go ahead and ask.

15 *Keisha:* Okay, the one you crossed out. So, I was going to say, because, couldn't you say AR, GR, MR and then say half of 550 and make that equal, since that's recreation and that half will be development?

16 *Mr. Taye:* Wait, wait, wait, say that one more time.

17 Keisha: Couldn't you take AR + GR + MR and whatever half of 550 is and do that and then do AD + GD + MD and that other half and then that could be one.

18 *Mr. Taye:* Anybody try to answer that?

19 *Ss:* I did not understand what she is saying.

20 *Mr. Taye:* She is saying, since this [AR + GR + MR +AD +GD + MD] all totals to 550, couldn't you simply takeAR + GR + MR and make that equal to whatever half of 550 is. Make that one equation.

21 *Emily:* But it won't be a constraint, then.

22 *Mr. Taye:* Could you do that?

23 *Ss:* Yeah.

24 *Emily:*I don't think so.

25 *Jovita:* But we don't know what AR, GR and MR are.

A discussion ensued and Mr. Taye asked the students what statement should be in problem for them to create the equation AR + GR + MR = 275 (half of 550). Some students responded and the discussion ended as follows

26 *Mr. Taye*: Or, the land devoted to recreation has to be 275. Was that ever stated?

27 *Ss:* No.

28 Mr. Taye: Okay, we can't assume that this [pointing to AR + MR + GR = 275] might be true. This [pointing to AR + MR + GR + AD + GD + MD = 550] is true, but we don't know if this [pointing back to AR+MR+GR=275] can be true.

Following this, the students began to identify the additional constraints. They tried to derive them from the six constraints they already identified. Mr. Taye asked several questions such as – what values can AR, MR take and what values they cannot take (for e.g., can AR be equal to 500, -10 and so on), to help students identify the additional constraints (i.e. AR, MR, GR etc., are all ≥ 0) that are independent of the constraints they had identified earlier.

29 *Mr. Taye:* Is there anything that I can't do where I still take them and it totals a 100 [referring to allocating values for AR, MR such that AR + MR = 100, one of the constraints is satisfied].

30 *Ss:* What?

31 *Mr. Taye:* It is tricky. I am trying to do this without giving it to you. So what you are telling me is that all these two numbers that you gave me, what was true about those two numbers?

32 *Ss:* They equal to 100. [They add up to 100]

33 Mr. Taye: They equal to 100, right? Are there two things that you put in this box that will still equal a 100 but it won't apply to this situation.

34 *Ss:* Negative. You can't have negative land. -5 and 105.

35 *Mr. Taye:* Okay, that [referring to the allocation -5 and 105] adds to 100, but is this possible.

36 *Ss:* No, can't have negative land.

37 *Mr. Taye:* Okay, you can't have negative land, so tell me another constraint. So what has to be true about the amount of land that I use for army recreation?

38 *Ss:* It has to equal 100. It got to be positive, greater than 0.

39 *Mr. Taye:* It has to be positive or 0, so it has to be greater than or equal to 0. So this is actually a constraint, so this has to be true.

.... [He asks the students to start writing the other 5 constraints. Students start discussing in their groups to complete the 6 additional constraints.]

40 *Sally:* [low voice to Mr. Taye] Why do we need the equation, obviously we know we don't have negative land, so why….[inaudible]

[Mr. Taye stops the smaller discussions, says "good question" and repeats the question to the entire class. Some students attempt to answer the question.]

41 *Mr. Taye:* Ok, I am not ready to answer that question. don't want to answer that yet. You are going to answer that question. We are going to answer the, I could easily answer that question right now. We are going to do some assignments where I want you to realize the answer to that question. So, for right now, just trust me and eventually you will realize, you will make sense of that and if it never happens I will make sense of it for you.

I end this questioning episode here and will focus on Keisha's question (15 & 17) and Sally's (40) in the next section. I include below Mr. Taye's comments on these questions.

Mr. Taye [on Keisha's question]: Again, on the surface that might make sense. However that is like a question that I want her to think about and know that it doesn't work that way. She should be able to understand how the constraints work, that the land can be allocated differently. I did not necessarily think this was a good or bad question, she has not made the connection between all those things, that all those things combined is 550. I don't know if to say that she is not collecting the information correctly, or not understanding what the limitations are, the open ended-ness of the problem for her to say so. I would expect her to know the answer to that question. I would not expect that question to be asked. Because intuitively that doesn't make sense, intuitively you would think, if you know, looked at what they tried to do, she should know better than that, you can't simply state that this divided by two and these things have to equal 550/2. I mean, I could be wrong, but I just did not feel like that was a uhm.. I think with a little bit more of thought she could have answered that question herself.

Mr. Taye [on Sally's question]: Because intuitively it would make sense that of course you don't have to do that. I guess it was a good question in the sense that, it made sense to ask that question. I am hoping that as we solve these equations algebraically, that she would be able to answer that question. She does not realize that how solving these equations algebraically are really connected to solving it graphically. If you don't include those constraints algebraically, that makes your range of solutions infinite and way more work. So it would have been too hard to articulate that without having gone through the process.

Analysis

Mathematical objects (for e.g., variables, constraints, equations) are an outcome of *reification* – of our mind's ability to perceive the result of processes as permanent entities in their own right within a symbol system (Sfard & Linchevski, 1994). Processes at one level become objects or products at another level, or mathematics acted at one level becomes mathematics observed at another (Pirie & Kieren, 1994; Resnick, 1992). There is a process-product duality in that these objects are not only the signifier but also the signified (of a higher level process) and follow rules and processes within the symbolic system. Reification increases manipulability by not having to constantly think about what is signified. It is this possibility that spurs structural thinking in algebra which is characterized by a broadening of the view, condensing information in concise symbolic notations as opposed to an operational outlook that dictates the actions to be taken to solve the problem at hand.

Several authors (Pirie & Kieren, 1994; Resnick, 1992; Sfard & Linchevski, 1994) have stressed the importance of two aspects of mathematical understanding, namely, a) development of a formal, symbolic system that can operate independent of constant referencing to the empirical thereby allowing increased manipulability and b) the flexibility to move between the empirical and formal perspectives, and to let go of the referents when working with symbols but to come back to them when needed. So while reification and the development of structural outlook is desirable, flexibility to move between the structural and operational outlook, and to fold back to the empirical context is equally necessary to prevent problems arising out of empty symbolisms and pseudo-structural conceptions (Sfard & Linchevski, 1994).

Mr. Taye considers Keisha's equation within the larger context of the problem (1). He recognizes the need to "address that in a minute" (2) and that for the purpose of this problem they "need to break this one big equation into three smaller equations" (3). While the students are working on a specific task of creating constraints, he zooms out from this task to consider how Keisha's equation fits the entire unit problem. He zooms back in and states a specific action that needs to be taken "for our purpose." Keisha however is not thinking of this equation from such versatile perspectives. She is focused on the task at hand. In fact when Mr.

Taye asks them to break the bigger equation into two smaller equations, her focus shifts to a more specific level – how to get smaller equations from this big equation. The problem context (the general view) is dropped and the focus is on how to "do that" (15 & 17).

He did not "think this was a good or bad question," and noted that "she has not made connections to all those things." "While on the surface it [her equation/question] might make sense," he expected her to "know better than that", to be able to "understand how the constraints work" and not "simply state that this divided by two and these things have to equal 550/2." He recognized that she did not connect the context to the process of getting the equations. But the difficulty of doing this and what is required to be able to do this is attributed to "collecting the information correctly, or not understanding what the limitations are, the open endedness of the problem."

Sally on the other hand seems unable to let go of the context (40). She wonders why this additional constraint is needed when it is obvious that negative land doesn't exist. For her, AR is strongly connected with the context – AR is a label for the land allocation, a signifier. She does not yet think of AR as a variable within a system of equations that needs to run on its own to provide a solution to the problem. Sally's question depicts the difficulty involved in developing the structural outlook and relating the formal system and the concrete situation. That one needs to create a set of equations (signifiers) representing the concrete context so that when we let go of the concrete context the formal system will act on these signifiers (which now become the signified) to produce a result valid for the situation.

Mr. Taye appreciated this question in class and chose not to give an answer. He wanted her to "realize and make sense of it herself," through the work in class. When I asked him why he considered this a good question, he responded "it made sense to ask this question." Mr. Taye recognized that Sally "does not realize that how solving these equations algebraically are really connected to solving it graphically," that she has not yet made this connection.

Mr. Taye's comments indicate that he expects students to ask questions such as Sally's. His expectation may indeed be a desirable one. What makes questions like Sally's good and perhaps even desirable and Keisha's question not so "intuitive"? Both Keisha and Sally's question highlight the difficulty of making connections between the formal system and the concrete context, developing a structural outlook from the operational one, and in letting go of the concrete system when required and yet being able to fold back when needed. Both Sally and Keisha's question arise out of the connections that are not obvious to them, but are obvious to Mr. Taye from his structural perspective.

Mr. Taye expects Keisha's question to be answered based on prior knowledge. Questions which "they could have thought about and answered themselves" are not necessarily expected to be asked. It is possible that questions that reflect students "thinking deeply", "making connections," and "extending their thinking" are ones that Mr. Taye sees as having the potential to make connections and develop a next level of mathematical understanding that is broader and more general. While asking such questions students are thinking at the boundaries of the structural and operational approaches. By asking such questions students are beginning to observe the processes they are working on and think about the processes so they can then "extend their thinking." If Keisha had asked why they were splitting the bigger equation into smaller equations, would that have shown evidence of her observing the processes they were working on instead of doing the processes and this perhaps would have been a question that "made sense to ask"?

When questions arose from a purely operational outlook Mr. Taye was left wondering and surprised as he expected his students to know the response to these questions or to be able to figure it out themselves with some thought. While he did respond and support students in their questions to "figure it out," he was disappointed at the lack of "higher order thinking questions" from students. Sfard & Linchevski (1994) propose that the operational outlook precedes the structural outlook but the latter does not develop immediately and is inherently difficult, requiring a transformation. In a later discussion with Mr. Taye, he wondered if memory loss, recall issues, or difficulty in application were causing this difficulty of students being stuck in the mode of doing and not thinking deeply.

Conclusion

In analyzing the student questions in the classroom discourse I have investigated the connection between one teacher's expectations of student questions and the levels of mathematical understanding evident in and required for such questions. The teacher has a broad view of the problem and what is needed to solve it, while the students are developing this broad view from the operational view that they currently have. Further, the teacher is able to move between the general and specific and is guiding the students from this place. The students however are focusing on *doing* the mathematics with this guidance.

The analysis shows questions that suggest students making sense of the mathematics from a structural outlook or beginning the shift from the operational to the structural outlook are considered desirable by the teacher. Questions that "make sense to be asked" are ones where students have begun observing the processes to reify them and develop the structural outlook. Questions that did not seem "intuitive" and did not satisfy the teacher's expectations were asked from a purely operational perspective. Furthermore, questions that students ask may reflect the difficulty in developing this structural outlook. It maybe desirable that all students begin asking questions which show evidence of the emerging structural outlook. But listening to *unexpected* questions that students ask can give insights into the difficulties they are facing in order to identify ways to support them in making these transformations, especially in algebraic thinking at the high school grades.

We know very little about how teachers interpret questions in relation to the kinds of mathematical understanding they want students to develop, and how they use student questions to inform their instructional practice. How do we better understand student questions in relation to developing mathematical understanding? How do other teachers view student questions and what kinds of questions do they expect? What does the teacher need to know and understand to support students in moving to the point where they can ask such questions? What tools do teachers have to make decisions about and assess student questions while they are teaching? These are some of the questions that need further investigation.

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