Creating a culture of enjoying mathematics: Notes from a small school*  
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Acknowledgement  
I would like to begin by thanking Prof Vijaykumar for giving me this opportunity to share with you some notes from a small school. I must also take this opportunity to thank my colleagues at Centre For Learning (CFL) who gave me tremendous support in preparing this talk.

After initially agreeing rather readily to make a presentation on my experience as a math educator, once I started jotting down my thoughts, I went through quite a struggle. Being trained as a mathematician to either make statements which can be proved, or to make it clear when one is stating a conjecture, I found myself preparing a talk where one cannot really make black and white statements, especially since we are talking about children and the elusive act of learning!

Just as I was despairing, I read a few research articles on math education and was very heartened to read that while the problems of math education are clear, and while there are several attempts to solve these, it is very hard for anyone to make simplistic cause and effect statements about the efficacy of these methods. And it was reassuring to learn that the problems faced in teaching and learning mathematics are universal, and that the issues and ideas I am going to share with you today are very central to many peoples’ concerns about math education.

So even though this talk will not be about conclusive statements, I hope it will give you all something to reflect upon.

Oh! Before I forget, I understand that I also need to thank Sri Atmaraman, Dr Shailesh Shirali and Prof K Subramaniam for conspiring together to land me in this situation in the first place!

Prologue  
If there is one thing we are all agreed upon as mathematicians or mathematics educators, it is that the state of mathematics education world over is far from satisfactory. There seem to be several problems with mathematics education. Many schools have very poor infrastructure and often an absent teacher. Even in schools with good infrastructure and teachers present, the curriculum is often dry and unimaginative and the textbooks even more so. Significant numbers of teachers have a poor understanding of their subject, and are often burdened with large classrooms and administrative duties. Not surprisingly, they seem unmotivated and do not seem be looking for creative solutions.

Coming to students, vast numbers seem to dread mathematics. There is great fear associated with learning mathematics and this fear often persists into adulthood. Moreover, students do not seem to achieve minimum learning standards.

The picture seems dismal and sometimes one has a feeling of hopelessness when looks at the problem at a global scale.

What is disappointing is the motivation behind most attempts to reform mathematics education. The main interest is to create mathematically competent humans who will become part of a ‘knowledge society.’ And the main goal of this knowledgeable society is to compete economically with other knowledgeable societies! We have seen that this approach has not really solved anything.

* The talk at NIME (South) 2011 in Kochi on 12 November, 2011 contained video tapes showing various aspects of learning mathematics at CFL and cartoons to illustrate the text. The cartoons have been removed for copyright reasons, and videotapes for confidentiality.
I mentioned that the global scenario is depressing; but at a small local scale the situation can be completely different! In today’s talk I would like to share with you our experience at Centre For Learning in creating an environment where children enjoy learning mathematics.

Centre For Learning (www.cfl.in) is a small school in Bangalore that was started in 1990 by a group of educators interested in the nature of true learning in all aspects of our lives. Many of us who work at CFL have been influenced by the teachings of the late philosopher J Krishnamurti.

Based on my experience as a teacher of mathematics for the last 25 years, 16 of them at CFL, I can say with some certainty that it is possible to create a learning environment where:

- children do not dread mathematics and have a healthy attitude towards learning it
- most of them are competent at mathematics (I will go into what I mean by this later) and many of them positively enjoy learning mathematics

One of our students who has just joined CFL from a good school in the city recently told her teacher, “Maths used to be my enemy, now it is my best friend.” When asked, the majority of our students would list mathematics as their ‘favourite subject’.

Before going any further, I would like to make a disclaimer. We are not churning out mathematicians by the dozen! And it’s not as if all our students are going to pursue mathematics or mathematically oriented subjects when they leave school.

My point is the following: I firmly believe that if we are going to make mathematics a core subject in primary education, then we owe it to our children that they find their experience of learning mathematics meaningful and enjoyable.

The question is, what is an enabling environment for the learning of mathematics?

In this talk I plan to go into the philosophy and practice of mathematics teaching at CFL. One of the strongest claims we can make about our students is that there are no math phobics among them. We have all heard the term ‘math phobia’, but the magnitude of it hit me when I came across a recent study [1] by Susan Picker and John Berry entitled ‘Investigating Pupils’ Images of Mathematicians’. The researchers asked children (between the ages of 12 and 13) from the US, UK, Finland, Sweden and Romania to draw a picture of “a mathematician at work”.

The images are quite graphic, for example showing a mathematics teacher with a gun threatening children to learn or else…What comes across clearly in the study is that children have no clue what mathematicians do for a living, have stereotypical images, experience themselves as helpless and their mathematics teachers as authoritarian and intimidating, and the learning process is experienced as coercion.

I am sure we can find countless stories from our own context where children experience learning mathematics as traumatic.

Given the fact that fear and learning do not go together, we feel that there has to be an environment of learning where the relationship between the teacher and student is based on mutual trust and affection. Where the tools for motivation are not fear, reward and punishment, competition and comparative evaluation. It is important that the child’s self-worth is not linked to her intellectual ability. The child should have the leisure to learn and find the process of learning meaningful.

I would like to add here that we also strongly believe that these conditions are absolutely necessary because when the opposite prevails—when fear and competition are the main tools of
motivation—they do great harm to children and eventually create uncaring and ultimately dysfunctional societies.

Creating a space where fear is not a motivating factor does not automatically eliminate fear in the child. At CFL we tackle this emotion head-on. Children are encouraged to be aware of their fears, to express them and to observe how they may be impacting their learning. In looking for the roots of this insecurity, one may find that it arises because the student has linked his self-worth with the ability to perform well. Often it arises because the student isn’t confident about his understanding. This block can be addressed both through the teacher putting more energy into explaining, and also the student himself working at it.

Creating the right environment to learning is necessary, but by no means sufficient, in meeting the challenges that the learning of mathematics throws up.

We have to understand the underlying beliefs and attitudes that teachers and children have about mathematics and the special pedagogical demands that mathematics makes. Let us look at beliefs to begin with.

**Epistemological beliefs**

In preparing for this talk I have become aware that since the 1980s many researchers are studying the link between beliefs and competence in mathematics. One paper states: “Fostering in students the acquisition of positive or availing mathematics-related beliefs is regarded as an important goal of mathematics education.”[2]

It would be a very interesting exercise for teachers to jot down their own beliefs about the nature of the subject they are teaching and why they are teaching it.

So let me put down the overarching ideas at CFL about the nature and teaching of mathematics:

- mathematics is deep and beautiful, and children must get a taste of this. Each child at his or her own level should experience the joy of understanding concepts and the pleasure of making connections.
- mathematics can be viewed in many ways, and children should be exposed to the different aspects of mathematics. No one view should dominate.
  - as an art form
  - as the language of nature
  - as a powerful tool to model our environment
  - as a tool for book-keeping in the world of commerce
- while problem solving is an important part of mathematics there is more to it than that. Children should be exposed to theory building as well as problem solving.
- mathematics is not a hoary body of knowledge that somehow should be painfully acquired; rather, it is a lively activity consisting of recognizing patterns, proposing conjectures and then proving these conjectures. Thus children should find learning mathematics meaningful and they should learn how to play with ideas, especially learning to recognize patterns and to represent their recognition of these patterns using mathematical notation
- there are many myths about mathematics that need to be constantly exposed. Here are a few:
  - I am stupid if I am bad at maths
  - The teacher knows all
  - Mathematics is all about ‘calculating’
  - There is only one way to do a problem
  - Mathematics does not afford experimentation and exploration
In the light of the above beliefs and aims perhaps it is important for me to share our curricular goals for mathematics at CFL. Hopefully you will find them reasonable and not eccentric! One point to keep in mind is: we strongly believe that all children up to grade 9 should be exposed to a common curriculum.

Our curricular goals are that children should:

- have a good sense of numbers and be comfortable enough to use them in practical everyday situations
- have a good spatial and geometrical sense and use these when required in daily life
- be comfortable with data its representation and interpretation
- master basic mathematical branches like arithmetic, algebra, geometry and trigonometry
- be able to learn and apply mathematical concepts in allied subjects
- learn the rudiments of logic and mathematical reasoning

So you can see that our curriculum aims at creating mathematically literate students or even mathematically ‘mature’ students. Interestingly it conforms to the goals that researchers in psychology and education seem to concur with. I will discuss these towards the end of my talk.

To fulfil these curricular goals, we need to choose appropriate material (the talk showed sample material to illustrate this point, from the Real Math series published by Open Court).

Nowadays, thanks to the Internet, educators can access freely available material which can provide ideas and supplements for the classroom.

**Delivery of the curriculum**

Apart from having clear curricular goals, it is important for us to consider the process children go through in order to achieve these goals. Is the overall framework of learning coherent? Is the experience of children meaningful? What is the daily transaction between teacher and child?

Let me share the pedagogical practices that we follow:

**Mathematics as a human endeavour:** every attempt is made to demonstrate that mathematics is a human endeavour. This is done by talking about the history of mathematics and stories of mathematicians, trying to explain if possible why humans might have needed/developed the mathematics that is being taught.

**Classroom environment:** the environment is kept light, yet rigour is not sacrificed for informality. The students’ collective attention has to be constantly held on what is being learned: keeping track of each child as the lesson progresses.

**Teacher explanations:** teachers spend significant amount of class time explaining concepts, and often get children to articulate what they have learnt. As and when possible, the teacher will try and connect what appear to be different parts of mathematics, so that the child’s learning is not compartmentalised.

“… to achieve successful mathematical understanding, we must go beyond telling children how to solve mathematical problems; we must reach a point where children are not only successfully producing mathematical solutions but also understanding why the procedures work and when the procedures are and are not applicable. This point may be reached by providing children with, and requiring that they contribute, to adequate explanations in their mathematics classrooms.” - Michelle Perry [3]

**Distributed authority:** students speak as much as, if not more than, the teacher. Children are called upon to articulate what they have learnt, as far as possible in precise language. Students spontaneously explain to each other what they have learned, and answer each other’s questions. Some comments may seem tangential or even irrelevant to a discussion, but, if followed up, often
yield unexpected connections and ways of understanding. The student who finds math easiest is not the star of the math class! Everyone feels equally important in class, in terms of attention, appreciation and affection.

**Mathematics as a social activity:** Students often work in groups and learn cooperatively. They engage in thinking together in solving problems and help each other to build the solution without a sense of competition.

**Teaching at specific age groups**
We now describe briefly special aspects of our mathematics teaching at each age group.

At the younger ages, as is perhaps common in many schools, a lot of the mathematics is done using concrete teaching aids and linking mathematics to concrete, everyday situations: activities like cooking, shopping and estimating weights and measures. Many concepts are taught and reinforced using games.

One very important feature of the junior school mathematics curriculum is the use of ‘Thinking Stories’. These are stories read to children where the context demands mathematical analysis and understanding (broadly defined). Many times the problems are open-ended and require the student to figure out whether she has all the facts to answer the question, or whether the given facts can lead to more than one solution. These sessions help enormously with solving the demon of school mathematics, ‘word problems’!

At the middle and senior school levels, the mathematics becomes increasingly formal and abstract. During these years, we also introduce projects and mini-investigations, a central aspect of mathematics teaching. Examples of projects we have done are tessellations, platonic solids, Pascal’s triangle, Fibonacci sequences, linkages, infinities, Mobius strip and Konigsberg bridges.

Projects are excellent avenues for cooperative learning and the enjoyment of mathematics as a social activity. They also help to dispel many of the myths I listed earlier that children have about mathematics. Projects offer an opportunity for students to experience topics not traditionally taught at school, and along with the mini-investigations demonstrate that mathematics indeed is the science (or art) of pattern recognition.

**Overall ambience**
We feel it is very important that children have contact with mathematics outside their classroom experience. As mentioned earlier, at the younger ages children learn about fractions, weights and measures by activities like cooking, estimating the heights of buildings and so on.

In the senior school mathematics classroom we have stored puzzles and games which many students play with when they are free, or before the teacher comes to class.

Children of different age groups present their project work to the whole school in Assembly. This allows for students to get a feel of what their fellow schoolmates are learning, and also to share in the excitement of learning something ‘cool’ in mathematics. For the children who make the presentations, there are many skills that they learn apart from the skills of public speaking: presenting mathematical ideas clearly to a wide range of audience, using correct mathematical terms, thinking on their feet to answer unanticipated questions.

Occasionally teachers also make presentations about mathematicians, the history of mathematics and book talks.

Every other year at CFL the whole school takes on the exploration of a single theme, culminating in a ‘Mela’ where our learning is shared with parents and friends of the school. While we have
had only one ‘Mela’ exclusively devoted to mathematics, mathematics has played a major role in other themes like Magic, Astronomy and even History.

**What does the research say?**

Perhaps it is a good idea at this point to take a pause and see what current research has to say about all this! I have taken the liberty to edit lightly for brevity. Here is an excerpt from [2].

There is currently a broad consensus among scholars in the field of mathematics education that becoming competent in mathematics can be conceived of as acquiring a **mathematical disposition**. Building up and mastering such a disposition requires the acquisition of five categories:

1. A well-organized and flexibly accessible domain specific knowledge base involving the facts, symbols, algorithms, concepts, and rules that constitute the contents of mathematics as a subject matter field.

2. Heuristic methods, i.e., search strategies for problem solving, which do not guarantee, but significantly increase the probability of finding the correct solution: for instance, decomposing a problem into subgoals.

3. Meta-knowledge. This involves knowledge about one’s cognitive functioning on the one hand, and knowledge about one’s motivation and emotions on the other hand (e.g., becoming aware of one’s fear of failure when confronted with a complex mathematical task or problem).

4. Positive mathematics-related beliefs, which include implicitly and explicitly held subjective conceptions about mathematics education, about the self as a learner of mathematics, and about the social context of the mathematics classroom.

5. Self-regulatory skills. These include the self-regulation of one’s cognitive processes (planning and monitoring one’s problem-solving processes) on the one hand, and skills for regulating one’s volitional processes/activities on the other hand (keeping up one’s attention and motivation to solve a given problem).

So, as you can see, at least according to these researchers, we are on the right track!

**Our challenges**

Like all educational environments we too have our challenges. In fact when external motivators such as fear, competition, reward and punishment are removed, we as educators confront the real issues of education.

Despite all the effort and thought put into our learning environment, we still encounter some children who resist learning. This problem with motivation is a human predicament (all of us face it), and our question is: how do we address it without resorting to the usual tricks?

One question we often ask ourselves is: are we adequately challenging the student who is ‘gifted’ in mathematics? These students enjoy our basic mathematics programme, with all the aspects I’ve explained in my presentation, and retain their love and sharp mind for mathematics till high school. They especially enjoy the projects where they feel stretched, since there is the scope to take on harder challenges. But there is no doubt that such students could have done much more, or gone much further, in the same time.

Finally, the biggest challenge of all: the learner is not a blank disc on which all knowledge can be burned! The learner influences his own learning. Despite the most conducive environment and the best of all efforts, the learner’s complexities (attitudinal, motivational, emotional) can limit his or her learning.
**Epilogue:**
I am sure the thought has crossed your minds that CFL being small is somewhat privileged in being able to create a culture of enjoying learning mathematics. Is our model replicable? In some of its crucial aspects, we believe, yes. There is research to show that the quality of academic learning need not be impacted by classroom size. We feel many of the ideas shared above can be implemented in so called mainstream schools, if only they are recognized as absolutely necessary for learning to take place. So who is the key agent of change then?

Clearly, it is the teacher! Ultimately, as I mentioned earlier, it is the daily transaction between teacher and child that is the core of all education. Unless this transaction is intrinsically meaningful, no educational reform will work. There is no way we can hide a teacher who is not proficient, motivated or imaginative.

Of course, in order for the teacher to function effectively, we need to empower teachers and give them some autonomy. Particularly primary school mathematics teachers, whose work of laying the foundations is arguably the most important in mathematics teaching. I would like to end this talk by quoting Liping Ma who conducted a landmark study [4] comparing Chinese school teachers and school teachers from the US, in the 90’s.

> “Elementary mathematics can be viewed as “basic” mathematics – a collection of procedures – or as fundamental mathematics. Fundamental mathematics is elementary, foundational and primary. It is elementary because it is the beginning of mathematical learning. It is primary because it contains the rudiments of more advanced mathematical concepts. It is foundational because it provides a foundation for students’ further mathematics learning.”

We all fall into the trap of thinking that teaching elementary mathematics is rather trivial. However, Liping Ma showed in her study that good teachers have a ‘profound understanding of fundamental mathematics’. This is something everybody involved with mathematics education should understand and promote.

**References**


[2] Unravelling the Relationship Between Students’ Mathematics-Related Beliefs and the Classroom Culture. Erik De Corte, Lieven Verschaffel ,and Fien Depaepe Center for Instructional Psychology and Technology (CIP&T), University of Leuven, Belgium
